Explaining Student Expertise with Mathematical Sense-Making

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Problem Solving in PER

- Understanding and teaching expertise in problem-solving
  (Larkin, McDermott, Simon, Simon, 1980; Heller et al., 1992; Huffman, 1997)

- Rubrics to assess problem-solving expertise

- Rubric scores should match student verbal explanations
  (Docktor and Heller, 2009)

Problem Solving Rubric

(Docktor and Heller, 2009)

- Useful Description
- Physics Approach
- Specific Application of Physics
- Mathematical Procedures
- Logical Progression

What Rubrics Miss

Rubrics can miss unidentified forms of problem-solving expertise.

The "Two Rocks" Problem

Suppose you are standing with two heavy rocks high up on a tall building. You throw one rock down with an initial speed of 2 m/s; you just let go of the other rock.

What is the difference in the speeds of the two rocks after 5 seconds - is it less than, more than, or equal to 2 m/s?

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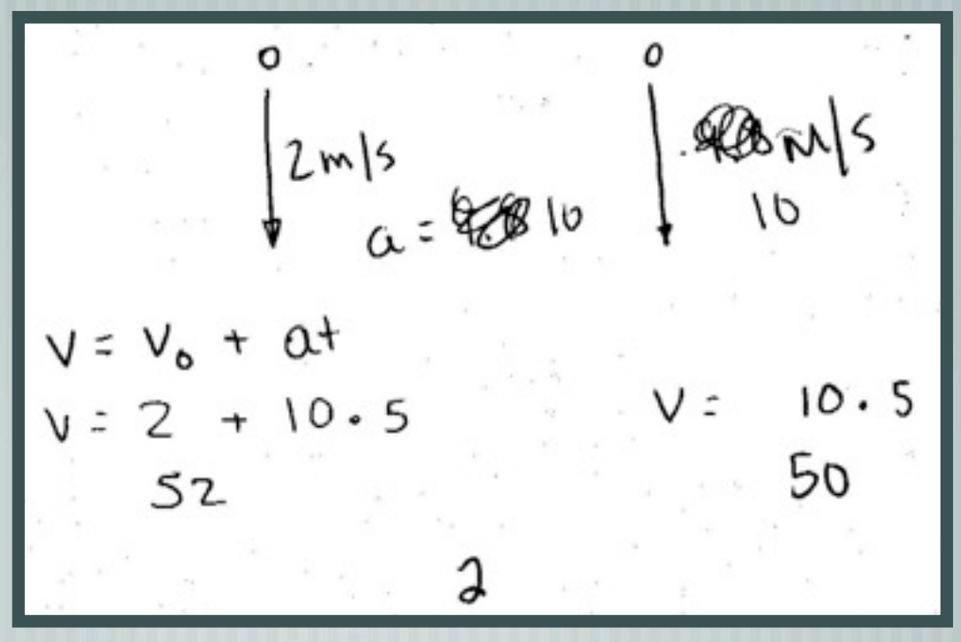
Alex works procedurally

- Draw a diagram and label with relevant information
- Use the equation: $v = v_0 + at$
- Plug in numbers for both rocks into the equation.
- Calculate the speeds and take the difference

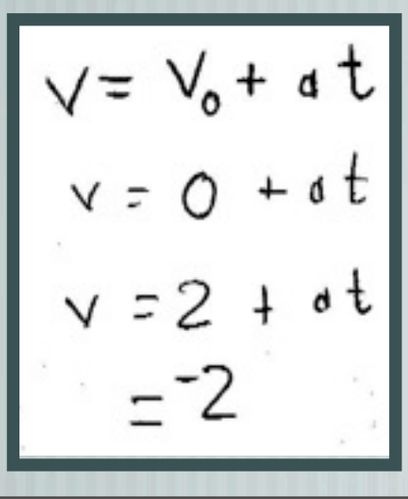


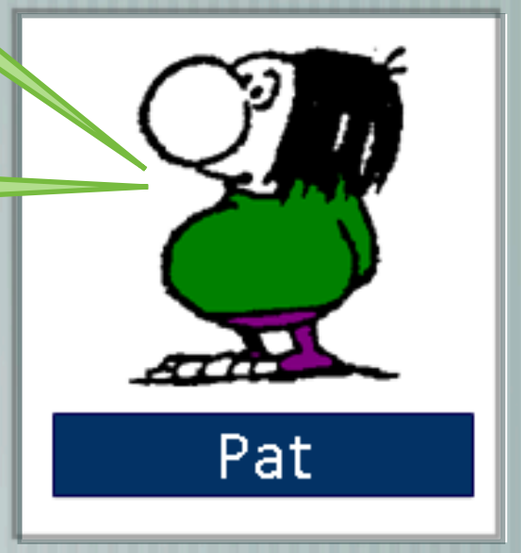
Student cartoon figures reproduced from: Allie, Buffler, Kaunda, Campbell, & Lubben,
First year physics students' perceptions of the quality of experimental measurements, *International Journal of Science Education*, 20(4), 1998, 447-459

Pat uses conceptual reasoning and math

"The initial conditions are off by 2 and then the velocities are changing at the same rate so that should mean they stay at 2."

"Even though it's a conceptual question, it's good to look at the equations and see how they behave in relation to the motion."





What Would the Rubric Say?

- Physics Approach + Specific Application of Physics:
  - Both use velocity equation and correctly insert values
- Mathematical Procedures:
  - Both execute correctly.
- Alex's and Pat's expertise with math in problem-solving is indistinguishable by this rubric.

Alex and Pat Use Math Differently

Both use the equation:

$$v = v_0 + at$$

Equation for plug-and-chug calculation

Equation supports conceptual reasoning

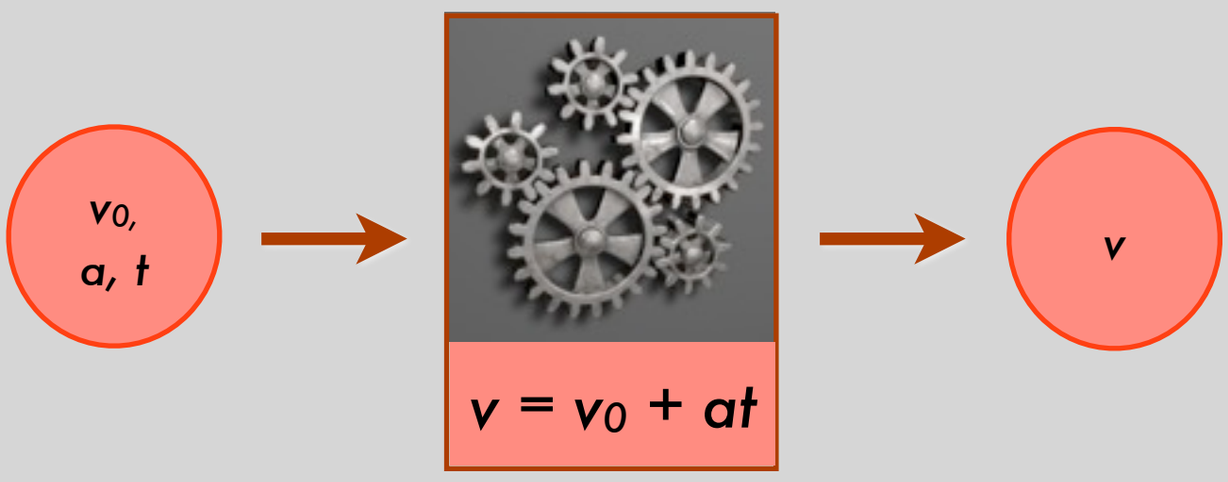
Explain "$v = v_0 + at$"

"You can find the velocity at any time if you have the initial velocity, the acceleration, [and] time."

"You start from the velocity you have in the beginning and you find out how the acceleration affects that velocity. Then that would be the significance of each term."

How "$v = v_0 + at$" is treated

Equation for plug-and-chug calculation



Equation supports conceptual reasoning

(Sherin, 2001)

Conclusions

- Two different ways mathematics is used
- Rubric misses connection between the equation and physical process
- Any rubric that catches this type of expertise would examine problem-solving as a whole.
- In the following talk, why this form of expertise is significant.