Explaining Student Expertise with Mathematical Sense-Making

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Problem Solving in PER

Understanding and teaching expertise in problem-solving
(Larkin, McDermott, Simon, Simon, 1980; Heller et al., 1992; Huffman, 1997)

Rubrics to assess problem-solving expertise

Rubric scores should match student verbal explanations
(Docktor and Heller, 2009)
Problem Solving Rubric

(Docktor and Heller, 2009)

Useful Description

Physics Approach

Specific Application of Physics

Mathematical Procedures

Logical Progression
Rubrics can miss unidentified forms of problem-solving expertise.
The “Two Rocks” Problem

Suppose you are standing with two heavy rocks high up on a tall building. You throw one rock down with an initial speed of 2 m/s; you just let go of the other rock.

What is the difference in the speeds of the two rocks after 5 seconds - is it less than, more than, or equal to 2 m/s?
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Alex works procedurally

- Draw a diagram and label with relevant information
- Use the equation: \( v = v_0 + at \)
- Plug in numbers for both rocks into the equation.
- Calculate the speeds and take the difference

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Student cartoon figures reproduced from: Allie, Buffler, Kaunda, Campbell, & Lubben,
Pat uses conceptual reasoning and math

“The initial conditions are off by 2 and then the velocities are changing at the same rate so that should mean they stay at 2.”

“Even though it’s a conceptual question, it’s good to look at the equations and see how they behave in relation to the motion.”

\[
\begin{align*}
\nu &= \nu_0 + at \\
\nu &= 0 + at \\
\nu &= 2 + at \\
\nu &= -2
\end{align*}
\]
What Would the Rubric Say?

Physics Approach + Specific Application of Physics:
- Both use velocity equation and correctly insert values

Mathematical Procedures:
- Both execute correctly.

Alex’s and Pat’s expertise with math in problem-solving is indistinguishable by this rubric.
Alex and Pat Use Math Differently

Both use the equation:

\[ v = v_0 + at \]

Equation for plug-and-chug calculation

Equation supports conceptual reasoning
Explain “\( v = v_0 + at \)”

“You can find the velocity at any time if you have the initial velocity, the acceleration, [and] time.”

“You start from the velocity you have in the beginning and you find out how the acceleration affects that velocity. Then that would be the significance of each term.”
How “$v = v_0 + at$” is treated

Equation for plug-and-chug calculation

Equation supports conceptual reasoning

$v_0, a, t \rightarrow v = v_0 + at \rightarrow v$

$v = v_0 + at$

Final Velocity = Starting Velocity + Change Due to Acceleration

(Sherin, 2001)
Conclusions

Two different ways mathematics is used

Rubric misses connection between the equation and physical process

Any rubric that catches this type of expertise would examine problem-solving as a whole.

In the following talk, why this form of expertise is significant.