Project overview

For engineers, effective use of mathematics is more than manipulating equations and applying algorithms; it involves *mathematical sense-making,* looking for coherence and meaning partly by translating back and forth between symbolic relations on the page and relations (causal and functional) in the world [1-11]. Mathematical sense-making is central to students' success with modeling and design [2, 8-10, 12]. Yet, many engineering students have trouble with it [13-15].

Typical engineering students first grapple extensively with mathematical descriptions of the world in the introductory physics courses they take as prerequisites for their majors. Those physics courses can forge or harden students' attitudes and approaches toward math. This project, a collaboration among the University of Maryland Departments of Physics, Mechanical Engineering, and Electrical & Computer Engineering, addresses two research questions:

1) What factors contribute to students' difficulties with mathematical sense-making?

2) Can redesigned introductory physics courses improve students' mathematical sense-making — and overall performance — in their later engineering courses?

Previous research in engineering, physics and mathematics education suggests that, to address (1), we must probe not just for mathematical skill deficiencies but also for students' lack of understanding of the relevant physics/engineering concepts, lack of ability or propensity to translate between formalism and real-world relations, and naïve beliefs about how to learn and apply mathematics [16-19]. We will tease these factors apart and explore interactions among them using multiple methods, including analysis of students solving challenging problems in small groups, recorded on videotape.

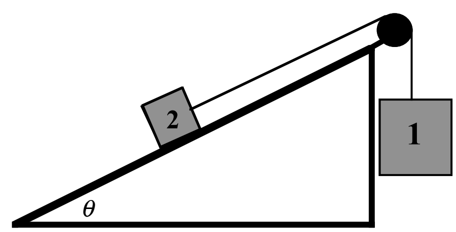
To address (2), we will draw on our previous work in the algebra-based introductory physics sequence for life science majors. There, we developed materials and teaching techniques designed to address students' intuitive "epistemologies," their beliefs about the nature of knowledge and how to acquire, organize, and apply it. Our courses produced substantially improved conceptual learning and mathematical sense-making [20]. In redesigning the calculus-based introductory physics courses for engineers, we will also be guided by our previous research on engineering students' epistemologies [13, 21, 22], which differ in some respects from those of life science students.

We will follow students from our redesigned physics courses, and from unchanged (control group) physics courses, into the Basic Circuit Theory class in Electrical & Computer Engineering and a Fluid Mechanics class in Mechanical Engineering. To assess students' mathematical sense-making, we will analyze their work on selected homework, exam, and specially-assigned problems. We will also videotape and analyze students working on problems designed to probe their mathematical sense-making, both in discussion section and in problem-solving interviews conducted outside of class. To investigate students' epistemological views about learning and using math, we will modify and administer a survey used in prior projects [22]. Finally, to see if students from our redesigned physics courses outperform the control group, we will look at exam scores and course grades in the targeted engineering courses.

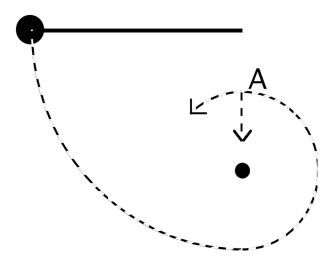
Rationale and goals

The importance of mathematical sense-making

According to engineering experts, educators, and education researchers, the effective use of mathematics cannot be reduced to a list of formal manipulations and problem-solving skills [1-11]. A solid skill set is essential, of course, but it must be integrated into a productive approach toward learning and using mathematics. In particular, it must involve *mathematical sense-making* [3, 4, 9, 10, 12], i.e., looking for meaning and coherence within the mathematical formalism itself and between the math and the system it describes [12, 18, 23]. A crux of this sense-making is the propensity and ability to translate back and forth between formal expressions on the page and causal or functional relations in the world. Mathematical sense-making rests on the belief that equations and operations *can* be made sense of, that equations "say" something [13, 18, 24]. By contrast, in their physics classes, many science and engineering majors express and enact the belief that equations aren't "supposed" to make sense, that they are mere problem-solving tools without important conceptual content, and that learning a given equation is a matter of memorizing the problem-solving algorithms in which it appears [13, 18, 21, 22, 24].



Co-PI Hammer, studying students in a calculus-based physics course required of Berkeley engineering majors, documented how students consider it appropriate to suspend their common sense in problem solving [13, 21]. "Roger," for example, was solving for the acceleration of the blocks in the modified Atwood's machine shown here. He found a sensible expression for the total force on the two-block system, $F = m_1 g - m_2 g \sin\theta$. In the next step of his solution, however, he set that force equal to the force on each individual block: $F = m_1 a_1$ and $F = m_2 a_2$. When he saw that this gave him a different acceleration for each block, he remarked that it seemed strange, but after checking his algebra and finding no mistakes, he said that he was "90% sure" he was right. This reflected a pattern in Roger's work, of treating mathematical problem solving as separate from tangible sense-making.



Other students displayed similar patterns. "Daniel" was solving a problem in which a pendulum swings and catches on a post: What is the highest placement of the post that still allows the pendulum bob to swing in a complete circle around it? Daniel's expression for conservation of energy omitted a term for the kinetic energy at the top of the swing, which led him to calculate $v = 0$ at point A. He remained confident in his solution, even though it makes no sense that the bob could complete its circle if it stops moving at point A.

Roger and Daniel both showed a pattern that was common among the students in Hammer's study, and in contrast to our findings in the algebra-based course: They were comfortable with mathematical manipulations, and they were willing to reason conceptually, but they did not insist that their mathematical work align closely with their conceptual reasoning. Two other subjects in Hammer's study, by contrast, persistently expected that mathematical calculations align with conceptual reasoning. One was "Tony," who spoke of mathematics as expressing ideas and of physics as "a matter of putting common sense into equations."

As it happened, Tony made the same mistake as Roger when he first tried the Atwood's machine problem, writing $F = m_1a_1$ and $F = m_2a_2$. But Tony immediately rejected the possibility that the blocks could have different accelerations. Pointing to his calculation, he explained that they "said the force is going to be … right here," indicating the smaller block, "and now I'm saying that's not true." He proceeded quickly to a correct solution, writing $F = (m_1+m_2)a$ and explaining that the force he calculated accelerates both blocks. *Tony's mathematical manipulation skills were not evidently better than Roger's*. Tony's more sophisticated approach to using math is what enabled him to learn the physics more deeply and to solve problems more flexibly. Unfortunately, Roger's and Daniel's stance toward mathematical problem solving was typical among students in the Berkeley course [13, 21]. Redish had similar experiences with students in the corresponding courses at Maryland [22].

We will test the hypotheses that (i) our redesigned physics courses help more students take a stance like Tony's, and (ii) in their engineering courses, students with a stance like Tony's outperform students with a stance like Roger's.

The need for a cross-departmental project

Roger and Tony illustrate why attempts to study and improve engineering students' mathematical sense-making cannot take place only in engineering courses. Because of time constraints, junior- and senior-level undergraduate engineering courses must generally presume that students already know how to "read" and "write" mathematics. By contrast, physics is where many engineering students first grapple extensively with mathematical descriptions of the world. Those physics courses can forge or harden students' attitudes and approaches toward using math in a science/engineering context [22]. If, as is commonly the case, physics courses reward or at least fail to penalize students like Roger who use mathematics without sense-making, then engineering professors face an uphill battle to help such students use math productively. By contrast, if physics courses develop students' skills, beliefs, and habits of mind involved in mathematical sense-making, then engineering professors can build upon that foundation. Regarding engineering students' facility with mathematics, physics courses are part of the problem but can become part of the solution.

For this reason, our project is a collaboration among the Departments of Physics, Mechanical Engineering, and Electrical & Computer Engineering. Moreover, the timing is perfect, because the Physics Department is currently initiating a redesign of the introductory sequence for scientists and engineers, realizing that it might not be serving engineering students as well as possible. We are working with the committee in charge of this process. See the attached letter of support from Prof. Hadley, Associate Chair for Undergraduate Education in Physics.

The need to look beyond skill deficiencies

Many instructors interpret students' difficulties with mathematics as a simple lack of basic skills, and they address those difficulties with remediation. We, too, expect to find and address some skill deficiencies. Previous research, however, challenges the adequacy of this simple diagnosis and prescription [13, 18, 22, 24, 25], as Roger and Daniel illustrate. Both of them were mathematically skilled: Roger scored a 5 on the BC Calculus AP exam, while Daniel, who left high school early, was concurrently enrolled in Calculus and getting an A. During interviews, neither showed any difficulty with computation. Nor could their difficulties be attributed to lack of content knowledge. In the example above, Roger noticed that his calculations gave a puzzling result; and Daniel did not need a physics course to know that the pendulum bob cannot be moving in a circle unless it is moving.

Rather, Roger's and Daniel's difficulties stemmed from a failure to connect knowledge of the world to mathematical expressions and calculations. Where Tony saw translating between mathematical relations on the page and causal/functional relations in the physical world — "putting common sense into equations" — as central to learning physics, Roger and Daniel saw this translation as unnecessary. To understand Roger's and Daniel's difficulties with mathematical sense-making, we must look beyond skill deficiencies to factors such their beliefs about what constitutes learning and using math in an engineering context [12, 16].

Indeed, a pure skill-deficiency explanation of students' mathematical difficulties is challenged by studies focused on high school students [26, 27], college math majors [18, 24], college life sciences majors in physics [19], and college engineering majors in math and physics [13, 21, 28, 29]. The materials and approaches we have developed in physics owe a particular debt to Treisman's work in college calculus [28, 29]. Studying African-American engineering majors at Berkeley, Treisman showed that their mathematical skill level upon entering the university did not correlate with their success in freshman calculus. One difficulty was *social*: Unlike their Chinese-American counterparts, the African-American students tended to study alone and to treat their social and academic lives as separate. Programs designed around skill remediation were not effective. Instead, Treisman designed a program to get African-American (and later, other underrepresented minority) students working collaboratively on *especially challenging problems*, which prompted them to address core mathematical ideas and to build intellectual community. Treisman argued that the program's documented success at raising achievement [30], subsequently replicated elsewhere, stemmed in part from using calculus problems focused on concepts and higher-order thinking rather than remedial problems focused on basic skills [29].

Prior work on which this project builds

"Learning How to Learn Science"

This project builds on *Learning How to Learn Science: Physics for Bioscience majors* (2000-2004, NSF ROLE, $1,200K; E.F. Redish, PI), which focused on our algebra-based introductory physics sequence. We implemented materials and teaching techniques designed not only to help students understand core concepts more deeply, but also to foster sophisticated epistemological beliefs about what constitutes knowledge and learning in science. We wanted students to view physics as a coherent web of ideas rather than disconnected pieces; to view learning as building their own understanding of the concepts and problem-solving techniques rather than just absorbing information and practicing routines; and to see equations as precise expressions of conceptual ideas rather than mere plug-and-chug problem-solving tools. This last item, helping students develop more sophisticated beliefs about and approaches toward the use of mathematics, is the focus of our proposed project.

The mathematical challenges for students in *Learning How to Learn Science* were not precisely the same as those facing engineering students. Unlike Roger and Daniel, the typical student in our algebra-based courses is not comfortable with mathematical manipulations, even simple ones. Still, we found that students' epistemologies play an important role in their approach to learning and problem solving [19, 25, 31, 32]. To address students' epistemologies in our algebra-based courses, we engaged them extensively in small-group work using materials focused on both conceptual and epistemological development; assigned homework and exam problems requiring reflection on the learning process and rewarding conceptual understanding and mathematical sense-making rather than formula-plugging; and made "learning how to learn" a focus of lectures and all other aspects of the course.

Our approach was successful. Before implementing these changes, our students achieved better-than-average conceptual gains but did not develop more sophisticated epistemological beliefs about learning and mathematical sense-making. After implementing the changes, when the same professor taught the same course, we achieved significantly higher conceptual gains as measured by the Force Concept Inventory [33], and we saw positive changes in students' epistemologies as measured by the Maryland Physics Expectations Survey (MPEX), a widely used beliefs/attitudes assessment [22]. For example, responding to the statement

> In this course, I do <u>not</u> expect to understand equations in an intuitive sense; they just have to be taken as givens,

more students disagreed at the end of the semester than at the beginning, in contrast to what happens in traditional classes [22]. Figures 1 and 2 compare our results to those of other courses.

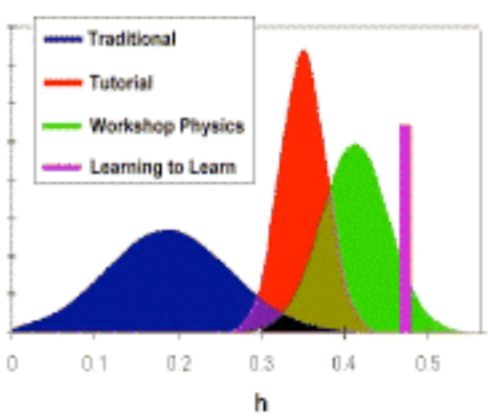


FIGURE 1: CONCEPTUAL GAINS [34]. *h* denotes the fractional gain on the Force Concept Inventory [33], a widely used standardized test of basic mechanics concepts. The fractional gain is students' actual gain (post test average – pre test average) divided by the maximum possible gain (100% – pre test average). *Tutorial* refers to one hour per week of collaborative group work using research-based materials. *Workshop Physics* refers to early small-school adopters of a curriculum in which students mostly do collaborative group work, with minimal lecturing. Histograms are constructed for each group and fit to a Gaussian, which is then normalized to unit area. The *Learning to Learn Physics* course achieved $h = 0.47$.

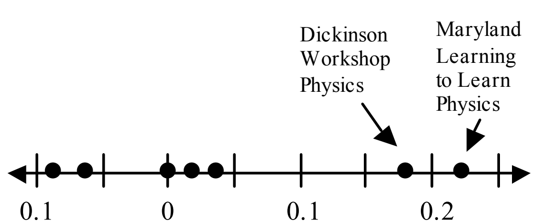


FIGURE 2: EPISTEMOLOGICAL GAINS. Fractional gains on the Maryland Physics Expectations Survey (MPEX) for seven institutions [22]. Despite being lecture-based, the Maryland course achieves slightly higher gains than a studio-based course in which students spend most of their time in collaborative active learning.

Crucially, we studied student reasoning during tutorial and problem-solving sessions, collecting and transcribing video data, which showed general consistency between the MPEX results and how students actually approached their learning and problem solving. For example, during collaborative group work in discussion sections and during group homework-solving sessions in the course center, as captured on video, we often saw students attempt to make sense of equations rather than just treat them as givens. (We have extensive evidence that students quickly forget about the camera's presence and behave similarly whether or not they are being taped.) In the proposed project, as in *Learning How to Learn Science*, we will use a combination of surveys, videotaped classroom interactions, performance assessments, and clinical interviews and problem-solving sessions to probe students' beliefs about and approaches toward the use of mathematics.

One form of analysis we'll perform comes from Bing's dissertation [35, 36] on upper division physics majors (*Learning the Language of Science: Advanced Math for Concrete Thinkers*, 2005-2008, NSF DUE/DTS, $300K, E.F. Redish, PI; *Toward a New Conceptualization of What Constitutes Progress in Learning Physics, K-16: Resources, Frames, and Networks*, 2005-2008, NSF ROLE, $700K, David Hammer, PI). Bing videotaped, transcribed, and analyzed pairs of students solving complex problems. He developed a coding scheme to characterize the students' epistemologies with respect to mathematics, finding a variety of ways in which students "frame" their use of mathematics. These frames include *Calculation*, taking mathematics as a procedure with the goal of producing a numerical or symbolic answer (as opposed to an explanation); *Unpacking math*, taking mathematics as a coherent body of knowledge with the goal of explaining the mathematical meaning of an object or operation; and *Mapping between math logic and physical world*, taking mathematics as an expression of conceptual meaning with the goal of connecting that meaning to the physical system. Bing found that students at any level often shift from one frame to another while solving a given problem. Graduate students, however, are more likely to be aware of and strategic about moving from one frame to another. Expert mathematical sense-making consists not of using *a* particular approach to math, but rather, of gaining "increasing flexibility and metacognitive control to switch approaches and expect coherence among them [36]." This insight informs our curriculum development, and Bing's coding scheme provides a starting point for analyzing how students approach mathematics in their Fluid Mechanics and Basic Circuit Theory engineering courses.

Materials and teaching techniques designed to foster mathematical sense-making

The following segment of an algebra-based tutorial is lower in level than the materials we will develop for the calculus-based physics courses in this project. Still, it illustrates one of the ways we foster productive student epistemologies for mathematical sense-making. Although the tutorial covers momentum and momentum conservation, we do not start with the formal definition of momentum. Instead, we start with common-sense ideas. Students work on these questions in groups of four, with a graduate-student teaching assistant helping as needed.

Figuring out the formula for oomph

An important physical quantity, the name of which we'll give later, corresponds to the intuitive idea of oomph. The more oomph something has, the harder it is to stop, and the more ability it has to knock other things over. Let's figure out the formula for oomph. If you already know the formula from a previous class, please "play along" and don't give it away; you'll learn something even if you already know the formula.

A. A small pebble and a larger rock are thrown at the same speed.

1. Which one has more oomph? Why?
2. The rock is twice as massive as the pebble. Intuitively, how does the rock's oomph compare to the pebble's? Is it twice as big? Half as big? Three times as big?

B. Picture two identical bowling balls, one of which is rolling faster than the other.

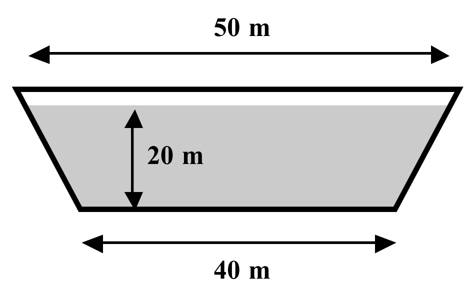
1. Which ball, the faster or slower one, has more oomph? Why?
2. The faster ball is exactly 7 times as fast as the slower one. Intuitively, how does the faster ball's oomph compare to the slower ball's oomph?

C. The physics concept corresponding to oomph is momentum. Building on your above answers, figure out a formula for momentum (oomph) in terms of mass and speed. Explain how the formula expresses your intuitions from parts A and B. (For historical reasons, physicists use the letter p for momentum.)

Even though the students in this course often find math to be intimidating, most have little trouble with parts A and B, and can successfully write and explain $p = mv$ in part C. (Vector considerations are introduced later in the lesson, and a follow-up lesson engages students in deriving the law of momentum conservation from Newton's 2nd and 3rd laws.) In conversations with the teaching assistants, many students say that although they saw $p = mv$ in high school, now they understand what the formula means and why it makes sense. That's the main point of the lesson; by "guessing" the formula, and by doing similar activities throughout the semester, students learn that physics equations carry conceptual meaning that they can make sense of. We will use similar activities, written at a higher mathematical level, in this project.

Another kind of mathematical sense-making activity engages students in thinking about and using a previously-learned equation. Here's an example we will pilot test this fall, after students have learned the hydrostatic pressure equation $p = p_0 + \rho gh$.

This trapezoidal dam is 20 meters high from the bottom to the water's surface. (The shaded section represents the part of the dam in contact with the water it traps.) The dam is 50 meters wide at the water's surface and 40 meters wide at the bottom, and it's 2.5 meters thick. Recall that the pressure at depth h is given by $p = p_0 + \rho gh$.



(a) Explain in terms a high school student could understand what that equation says and why it makes sense.

(b) A student, asked to find the overall force exerted by the trapped water on the dam, reasons as follows: "The force is the pressure times the dam's area, $A = 900\ \text{m}^2$. And the pressure is just $p = p_{\text{air}} + \rho_{\text{water}} gh$, with $h = 20$ meters. So, I'll calculate that p and plug it into $F = pA$." Does that approach overestimate, underestimate, or correctly estimate the force? Explain.

(c) Another student suggests modifying the strategy from part (b). "To make that strategy work, you just need to use the average pressure on the dam, not the pressure at the bottom. So, plug h = 10 meters instead of h = 20 meters into $p = p_{\text{air}} + \rho_{\text{water}} gh$, and then use that average pressure in $F = pA$." Does this modified approach work? If not, does it overestimate or underestimate the actual force on the dam? Explain.

(d) Now use calculus to find the force exerted by the trapped water on the dam. Explain why you set up the integral in the way you did.

(e) The problem-solving approach of parts (b) and (c) can work if you use the right h. Starting with your part (d) answer, figure out the right h. Explain in common-sense terms why this h makes sense, in light of the troubles with the h's used in parts (b) and (c).

(f) If the dam were rectangular rather than trapezoidal, would the approach of part (b) or (c) work? If so, which h would you use, and why? Would an integral also work? Explain.

This example encourages mathematical sense-making by having students think about what information is relevant vs. irrelevant (e.g., the dam's thickness) instead of providing only the needed numbers; by asking what the relevant equation "says" (part a); by considering multiple approaches (parts b-e); by explaining the reasoning behind a given calculation (parts d and e); and by reflecting on why a given strategy works or doesn't work (parts b-f).

The redesign of our physics courses extends beyond using new materials and engaging students in collaborative active learning during discussion sections. We also emphasize active learning and mathematical sense-making in lecture, building on Sokoloff & Thornton's Interactive Lecture Demonstrations [37] and Mazur's Peer Instruction technique [38]. For example, we regularly have students "find the sense in the equation," i.e., extract the physical meaning from what some students initially see as a barrage of symbols. Students discuss their ideas first with their lecture neighbors (nearest seatmates), then with the whole class, generating productive explanations the professor often carries forward into later lectures. Another technique we use starts with the professor posing a challenging multiple-choice question designed to bring out common conceptions and misconceptions. Students talk with their neighbors, then use personal response systems (clickers) to vote. The system anonymously compiles students' responses and displays a histogram for all to see. A relevant lecture demonstration and whole-class discussion follow, emphasizing the reconciliation of physics concepts with everyday ideas and experiences when possible. It's not enough to understand why the right answer is right; we also want students to explain to each other why the wrong answers are wrong, and to see the value of grappling with alternative points of view and addressing each others' arguments. These techniques foster a classroom culture of sense-making that leaks over into how they approach collaborative group work in discussion section and homework in the course center.

Project details

The project consists of three interacting threads: (1) Curriculum development and implementation, (2) Research into the causes of students' difficulties with mathematical sense-making, and (3) Research and evaluation about the effect of our redesigned physics courses on students' mathematical sense-making and overall performance in their later engineering courses.

Thread 1: Curriculum development & implementation

Two cohorts of students will pass through our redesigned three-semester introductory physics sequence for scientists and engineers. The first cohort will take Physics 161 in spring 2009, Physics 260 in fall 2009, and Physics 270 in spring 2010. The second cohort will pass through a semester later: Physics 161 in fall 2009, Physics 260 in spring 2010, Physics 270 in fall 2011.

Materials development and pilot testing will begin immediately, in fall 2008, and continue for the first year and a half of the project. See the Timeline table on page 12 for a semester-by-semester breakdown. Pilot testing will occur in two venues: (i) lecture and discussion sections of the current versions of those courses, and (ii) our "lab," where we pay small groups of students to get videotaped working through materials, an approach we have used successfully in the past. Note that we will not be developing materials from scratch. In addition to our algebra-based materials from *Learning How to Learn Science*, which will serve as first drafts in some cases and as ideas in other cases, we have large collections of calculus-based materials created in other projects, including Redish's earlier work on Maryland's Physics 161/260/270 (e.g., as part of *Activity Based Physics: Curricula, Computer Tools, and Apparatus for Introductory Physics Courses,* 1995-1998, NSF DUE, $1800K; Priscilla Laws, PI), Hammer's course materials from his current teaching of those same classes, and Elby's earlier work on the equivalent courses at UC Berkeley. Much of our development will consist of choosing the materials that best foster mathematical sense-making, refining them to accentuate and make explicit the mathematical sense-making agenda, and integrating them into a coherent whole — tutorials, homework, lecture examples, etc. — such that mathematical sense-making suffuses the courses.

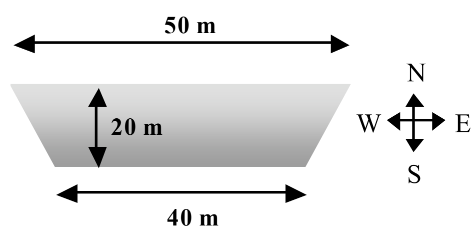
Another design principle of our physics course overhaul is to make explicit connections with students' engineering course work. For instance, students in Physics 161 will have taken ENES 100, a required freshman Introduction to Engineering Design class in which students design and build a hovercraft. Physics 161 will include lecture examples and homework problems to engage students in mathematical sense-making around specific concepts, equations, and design decisions they encountered while creating their hovercraft. Conversely, we will modify some of the discussion-section and homework problems in the targeted Engineering classes, Fluid Mechanics and Basic Circuit Theory, to reinforce and build upon the mathematical sense-making introduced in Physics. For example, a variant of the trapezoidal dam problem shown above could appear as a discussion-section or homework problem in the first unit of Fluid Mechanics. In order to assess students' mathematical sense-making and understanding of the material (as part of Thread 3), we will develop and pilot test similar problems, though with less scaffolding, for use on Fluid Mechanics and Basic Circuit Theory exams.

Thread 2: Research into students' difficulties with mathematical sense-making

Previous work has identified some of the factors contributing to students' competencies and difficulties with mathematical sense-making, as discussed above: skills, epistemological beliefs, propensity and ability to translate between formalism and reality, and so on [13, 14, 18, 21]. As Cardella [1] notes, however, little empirical work has explored the contributions and interactions of these factors, and perhaps new ones that arise, in engineering courses. (Cardella's research [1, 2], on which we will build, is a groundbreaking exception.) Because this is largely uncharted territory, we will do exploratory research, "tinkering" to build and refine analytical tools and forging hypotheses to test in future studies. Our goal is to make enough progress to (i) inform our curriculum development and implementation, and (ii) make contributions to the engineering education research literature — case studies, survey results, analysis of students' written and videotaped work, and promising hypotheses for future exploration — that spur us and other researchers to continue pursuing this line of inquiry.

In one line of work, we will try to isolate factors contributing specifically to *engineering* students' difficulties with mathematical sense-making. Hammer's studies of Roger and Tony illustrate one technique we'll use [13, 21]: By interviewing students several times, engaging them in multiple problem-solving tasks, he isolated epistemological beliefs as a main cause of Roger's difficulties and Tony's strengths. Video analysis of small-group problem solving is another, because in many cases when a group gets stuck, their discussions help to reveal why [19, 25, 35]. A third technique, which we can easily use on hundreds of students, involves *paired problems*. For example, in the trapezoidal dam problem presented above, suppose some students cannot set up and evaluate the integral to find the force exerted by the trapped water on the dam. We could give those same students a new problem formally equivalent to the dam problem:

> In the trapezoidal yard shown here, the grass density (grass blades per unit area) gets larger as you get farther from the north edge of the yard, because the south part gets more sunlight. The grass density is $\sigma = \sigma_0 + Cy$, where σ_0 and C are constants and y is the distance south from the north edge. If that density were constant, the total amount of grass is the yard would just be σA, where A is the yard's area. But here, you must use an integral to find the amount of grass in the yard. Set up and evaluate that integral.



If students do much better on this problem than on the dam problem, then we might have evidence that something about the fluids situation — conceptual difficulties with hydrostatic pressure, or troubles relating pressure to force, or intimidation by the complex fluids scenario — contributes heavily to students' difficulties in that case. By contrast, if students get stuck on the grass problem in the same way they get stuck on the dam problem, we might have evidence that the difficulty stems from skill deficiencies or from lack of ability or propensity to translate between physical scenarios and mathematical formalism. Of course, we cannot infer too much from one set of paired problems. Administering several such pairs, and using the other research methods discussed above, will enable us to make progress toward understanding which factors contribute most heavily to students' competencies and difficulties with mathematical sense-making in engineering.

Emerging results from year 1 will inform our research in year 2. If we find, for example, that Roger-like epistemological beliefs contribute widely to students' difficulties with mathematical sense-making, we will expand our epistemological survey [22] to explore students' beliefs in more detail, and we will devote more resources to adapting Bing's coding scheme (discussed above [35]) to characterize how engineering students frame their use of mathematics when solving problems.

Thread 3: Research/evaluation on effectiveness of redesigned physics courses

Every semester that we teach the redesigned Physics 161, 260, or 270, a second lecture section of the redesigned course will be taught by a physics professor who wants to "apprentice" with these materials and teaching techniques. Our interactions with that professor will include weekly meetings and observations of each others' classes. This second lecture section will enable us to evaluate whether the redesigned courses produce positive outcomes when taught by professors who lack previous experience in these teaching methods. So, instead of a treatment group (students taking the redesigned physics courses taught by us) vs. a control group (students taking unreformed physics courses taught by other professors), our experimental design is more complicated though more reflective of real life; different students will experience different degrees of treatment, depending on how many of their introductory physics courses are redesigned vs. unreformed and on the level of experience of the professors teaching their redesigned courses. We hypothesize that higher degrees of treatment will correlate with better mathematical sense-making and overall achievement in later engineering courses.

One batch of control data will come from students taking a separate, unreformed lecture section of Physics 161 in spring 2009 and forward through Physics 260 and 270. There may be some selection effects, however. For example, students who are aware of Hammer's reputation for nontraditional teaching may choose to take his class, or to avoid it. Scheduling conflicts might also push certain groups out of particular lecture sections. Fortunately, a "pure" control group will come from students who begin their physics sequence before spring 2009, i.e., before the redesigned courses are offered. In year 3 of the project, students taking the targeted Engineering courses (Fluid Mechanics and Basic Circuit Theory) will come from "pure" control, other control, and treatment groups, allowing direct comparison of overall course performance, exam performance, and performance on particular questions designed to assess mathematical sense-making. However, we can also compare control-group students from year 2 to treatment-group students from year 3, by analyzing performance on particular exam and standardized lecture questions that we reuse, and by comparing epistemological survey results.

The questions we use to compare students, and the rubrics we use to evaluate the quality of mathematical sense-making, will be developed and pilot tested during year 1 of the project.

To help us interpret our results from this thread, we will also use data from Thread 2 — most notably, video of students solving problems in small groups — to look for differences between pure control-group students and high-treatment students in the style, stability, and articulateness of their mathematical sense-making. This exploratory work will inform research and development in future projects, the first of which (we hope) will focus on students in engineering design courses making use of computational environments.

Timeline

<table>
<tr><th rowspan="2"></th><th colspan="2">THREAD 1
Redesigned physics curriculum*</th><th rowspan="2">THREAD 2
Research on student difficulties with math sense-making</th><th rowspan="2">THREAD 3
Research on effectiveness</th></tr>
<tr><th>DEVELOP</th><th>TEACH</th></tr>
<tr><td>Fall '08</td><td>PHYS 161</td><td></td><td rowspan="6">Data collection in the targeted engineering courses (Fluid Mechanics and Basic Circuit Theory)

Analysis is ongoing</td><td rowspan="3">Problem & survey development, pilot testing</td></tr>
<tr><td>Spring '09</td><td>PHYS 161
PHYS 260</td><td>PHYS 161</td></tr>
<tr><td>Summer '09</td><td>PHYS 260
PHYS 270</td><td></td></tr>
<tr><td>Fall '09</td><td>PHYS 260
PHYS 270</td><td>PHYS 161
PHYS 260</td><td rowspan="2">Control group data collection</td></tr>
<tr><td>Spring '10</td><td>PHYS 270</td><td>PHYS 260
PHYS 270</td></tr>
<tr><td>Summer '10</td><td></td><td></td><td>Analysis</td></tr>
<tr><td>Fall '10</td><td></td><td>PHYS 270</td><td rowspan="3">Thread 3 becomes main focus</td><td rowspan="2">Control & treatment group data collection</td></tr>
<tr><td>Spring '11</td><td></td><td></td></tr>
<tr><td>Summer '11</td><td></td><td></td><td>Final analysis</td></tr>
</table>

**Minor modifications to the engineering courses will take place over the first year of the project.*

Expected measurable outcomes

Effectiveness of the redesigned physics courses

At the $p < 0.05$ level, we expect higher degrees of treatment to correlate with better outcomes on

- Selected exam, test, and other questions designed to assess mathematical sense-making
- Epistemological beliefs assessment (survey) about the use of math in engineering
- Overall exam performance (comparisons possible for year 3 control groups only)
- Overall course grades (comparisons possible for year 3 control groups only)

The relevant statistical tests are multiple regression and ANOVA for outcomes measured in terms of continuous and discrete variables, respectively.

Research on causes of students' difficulties with mathematical sense-making

By the end of the project, we expect to have

- written at least four papers accepted for publication on this topic, with two more in progress.
- presented at least six talks to conferences attended by engineering educators, and at least three more at other STEM education research conferences.
- received a rating of "very good" or "excellent" from our Advisory Board (external evaluators) regarding the quality of our research on this topic. See next section for details.

Project evaluation

Since this project blends curriculum development with basic research in engineering education, a professional evaluation firm is unlikely to possess the needed specialized expertise. Instead, we have assembled an Advisory Board of experts in engineering education, cognition and learning, and mathematics education. They will evaluate and help guide the project. Specifically, each year, they will visit for two days to scrutinize our ongoing research; observe our physics and engineering classes, in some cases talking with students informally; and consult with us on ongoing course refinement and project management. In years 1 and 2, the Board will write a report to NSF and to us detailing strengths and weakness and suggesting improvements. At the end of year 3, the Board will write a cumulative, summative evaluation, including an overall rating of the quality of our research about students' mathematical sense-making.

Our Board members are extremely qualified to carry out these tasks:

KARL SMITH, Professor of Civil Engineering and of Engineering Education at U. Minnesota and Purdue, respectively, has devoted his career to building research capacity in engineering education. His national awards include the Distinguished Service Award, Educational Research and Methods Division, American Society for Engineering Education; the Carlson Award for Innovation in Engineering Education, American Society for Engineering Education; and the Outstanding Contributions to Cooperative Learning Award, Cooperative Learning Special Interest Group, American Educational Research Association. Editor-in-chief of *Annals of Research on Engineering Education*, and national advisory board member for the National Academy of Engineering Center for the Advancement of Scholarship on Engineering Education, he has written eight books, including *How to Model It*. He is currently co-PI of two NSF Centers for Learning and Teaching: the Center for the Advancement of Engineering Education (CAEE) and the National Center for Engineering and Technology Education (NCETE).

JAMES PELLEGRINO, co-director of the University of Illinois interdisciplinary Learning Sciences Research Institute and formerly Dean of Vanderbilt's Peabody College of Education and Human Development, is a nationally recognized expert in assessment, psychometrics, and cognition and learning. He has chaired or co-chaired several National Academy of Sciences/National Research Council study committees, including the NRC/NAS Study Committee on Learning Research and Educational Practice, which issued a widely-read report that he co-edited called *How People Learn: Bridging Research and Practice*. He co-chaired the NRC/NAS Study Committee on the Foundations of Assessment, which issued the influential report *Knowing What Students Know: The Science and Design of Educational Assessment*. A lifetime member of both the National Academy of Sciences and the National Academy of Education, he has authored over 250 books, articles, and study reports, and has supervised several large-scale research and development projects funded by NSF, NIH, and the US Department of Education.

ANN RYU EDWARDS, Assistant Professor of Curriculum & Instruction here at Maryland, is an expert on mathematical cognition/learning and on equity in mathematics education. Her dissertation advisor was Alan Schoenfeld, whose pioneering studies of mathematical metacognition and sense-making helped to inspire the research questions and methods of this study. Ann has extensive experience teaching the calculus courses taken by engineering majors.

HAYDEN GRIFFIN is Chair of the innovative Department of Engineering Education at Virginia Tech. An award-winning teacher (2006 Diggs Teaching Scholar), he has directed Virginia Tech's freshman engineering program for 11 years, overseeing major revisions and assessments, leading to improved student satisfaction. Over the past few years, he has transformed the Department to offer graduate degrees in engineering education research.

Personnel

The project team consists of engineers recognized for excellence in teaching and curriculum development, and members of Maryland's Physics Education Research Group, a national leader in curriculum development, teaching, and cognitively-grounded education research. The physics education researchers in this project have connections to engineering education as well.

EDWARD (JOE) REDISH (PI), Professor of Physics and Affiliate Professor of Curriculum & Instruction, is a national figure in physics education research. The recipient of local and national awards for teaching and education research (NSF Director's Distinguished Teaching Scholar Award, 2004; American Association of Physics Teachers Millikan Medal, 1998; University of Maryland Distinguished Scholar Teacher, 2006; Maryland State Board of Regents Teaching Award, 2007), he wrote *Teaching Physics* and he founded and edited the first journal devoted solely to physics education research (*American Journal of Physics Supplement: PER)*. He serves on advisory boards of the National Academy of Engineering Center for the Advancement of Scholarship on Engineering Education, and of Virginia Tech's Department of Engineering Education. He co-authored a paper with Karl Smith for *Journal of Engineering Education* [11].

DAVID BIGIO, Associate Professor and Director of Undergraduate Studies in Mechanical Engineering, is an award-winning teacher (Poole Senior Faculty Teaching Award, 2002-03; Center for Teaching Excellence Lilly Teaching Fellow, 1996-97). Involved with curriculum development for 12 years, he spearheaded the redesign of core engineering classes including the Engineering Project, Fluid Dynamics and capstone Engineering Design courses. He led the BESTEAMS project (*Building Engineering Student Team Effectiveness and Management Systems,* NSF CCLI, 2005-2008), which studied characteristics of effective teams in engineering and created curricular modules — now published as a book — for team building.

WES LAWSON, Professor and Associate Chair for Undergraduate Education in Electrical and Computer Engineering, has won numerous teaching awards ("Keystone Professor" in Engineering, 2006; Kent Outstanding Teaching Award, 1991; Corcoran Memorial Teaching Award, 1989) and helped to develop eight new engineering courses. He is currently PI of an NSF Research Experiences for Undergraduates (REU) site (*Training and Research Experiences in Nonlinear Dynamics*, 2006-2009, $180K), and was previously PI of another REU site (2003-2006) and co-PI of *Research Internships in Telecommunications Engineering (RITE) Site* (1998-2001, NSF EEC, $550K).

AYUSH GUPTA (PD) is a post-doctoral research associate with the Physics Education Research Group. His Ph.D work in Electrical & Computer Engineering here at Maryland focused on analytical and computational modeling of intense laser-plasma interactions. He currently works on two projects, one focused on students' ontological views about how to categorize entities and processes and the effects of those views on students' reasoning, the other focused on feedback loops and other couplings between students' nonverbal behaviors and the substance of their reasoning during collaborative group work.

DAVID HAMMER, a Professor with a joint appointment in Physics and in Curriculum & Instruction, is a well-published researcher of students' epistemological beliefs, starting with his dissertation work on engineering majors taking introductory physics courses at UC Berkeley. Also a nationally recognized scholar on cognition and learning (e.g., he is Associate Editor of *Journal of the Learning Sciences*), he has years of experience teaching the calculus-based introductory physics courses at Maryland and serving as PI of numerous research and curriculum development projects (e.g., *Toward a new conceptualization of what constitutes progress in learning physics, K-16*, 2005-2008, NSF/ROLE, $800 K).

ANDREW ELBY, a research scientist in Physics and in Curriculum & Instruction, began his work in education at UC Berkeley, helping to overhaul the calculus-based introductory physics courses aimed at scientists and engineers, and later serving as Assessment Coordinator for an engineering education research project (sponsored by General Electric) studying the introductory science and math courses taken by engineering majors. At Maryland, his research and curriculum development projects have focused on studying and changing students' epistemological beliefs (e.g., as PI of *Helping students learn how to learn: Open-source physics worksheets integrated with TA development resources*, 2004-2008, NSF CCLI, $400K).

Sustainability and dissemination

Local: In Physics, the research team is coordinating with the Physics 161/260/270 oversight committee, and professors will "apprentice" with our materials and teaching methods, as described above. For these reasons, successful aspects of our course modifications are likely to persist. Similarly, Prof. Lawson's status as Associate Chair for Undergraduate Education in Electrical & Computer Engineering, and Prof. Bigio's status as Director of Undergraduate Studies in Mechanical Engineering, will help ensure the institutionalization of successful modifications to Fluid Mechanics and Basic Circuit Theory.

National: Prof. Redish's connections to the engineering education research community will expand to include other Physics Education Research Group members, who will begin presenting at engineering education conferences and publishing in appropriate journals. The research team's proven record of creating and publishing curricular materials, and of publishing education research, will further help to ensure widespread exposure. We will make materials and research results available online at the National STEM Digital Library and on other sites. Finally, the research team has a proven record of following up projects of this type with NSF/CCLI and other grants to refine and disseminate materials nationally.

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