

Improving Engineering Students' Mathematical Sense-making

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Mathematical Sense-making

Looking for coherence and meaning partly by translating between symbolic relations on the page and relations (causal and functional) in the world

- not just manipulating equations, applying algorithms
- central to students' success with modeling and design

Research Objectives

Diagnose student difficulties with mathematical sense-making

- Weak conceptual understanding?
- Inadequate math skills?
- Epistemological issues?
 - Belief that formal and everyday thinking don't connect
 - Belief that equations are just problem-solving tools

Determine impact of reformed physics courses on student reasoning in engineering courses

- Curriculum focused on changing students' approaches to learning*
- Follow students from physics course into Basic Circuits Theory and Fluid Mechanics

Sources of Data: Video of students solving problems alone and in small groups

Sources of Data: Exam answers and scores, survey responses, course grades

Example HW Problem

Standing on a cliff, I take one rock and throw it straight up at a speed of 30 m/s. I take another rock and throw it straight down at 30 m/s. Suppose the cliff is 50 meters high.

(a) **Just based on common sense**, which rock would be moving faster when it hits the ground, 50 meters below? What's the reasoning for that?

Can't just use equations!

(b) Now find an answer based on the kinematics of constant acceleration: Find $x(t)$ and $v(t)$ for each of the rocks, and find their respective speeds when they hit the ground 50 m below the point of release.

Math skills are still important

(c) Did your answer to (b) agree with your answer to (a)? If not, try to reconcile the contradiction: Figure out what it is about the reasoning in part (a) or part (b) that doesn't work. Get it all to make sense!

Prompting for coherence in math & reasoning

• E. F. Redish & D. Hammer. (Accepted for Publication). Reinventing College Physics for Biologists: Explicating an Epistemological Curriculum, *American Journal of Physics*
 ** pseudonym used
 Project Website: <http://umdperg.pbwiki.com/Engineering+Education+Project>



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Case Study: Making sense of hydrostatic pressure equation

Jim** makes a mistake, but sticks to his answer.

$p = p_{at\ top} + \rho gh$
 Is pressure at a depth of 5 meters greater than, less than, or equal to the pressure at 7 meters?

h is negative because it is downward
 $p = p_{at\ top} + \rho g(-5)$ vs.
 $p = p_{at\ top} + \rho g(-7)$
 Pressure at 5 meters is greater.

His mistake does not stem from faulty physical understanding of pressure...

How would a friend who does not know physics answer this?

Someone reasoning with common sense would say pressure is greater at 7 meters, based on experiences under water.

...or from entrenched difficulties with co-ordinate systems:

Interpreting and using $v = v_0 + at$ for a problem about dropped object

Jim productively chooses "down" as the positive direction

With just a little help he could finally resolve his difficulty.

What about the sign of g in that equation?

g should also be negative. ...ooooo...the two negatives would cancel. And so pressure at a depth of 7 m would be greater.

Although Jim is not surprised that math and everyday reasoning "agree"...

[After inconsistency is resolved]
 What do you think now?

So our common perception is correct. Pressure does increase with depth. It makes more sense now.

...his difficulty stems from a tendency not to try to resolve inconsistency between formal and everyday knowledge

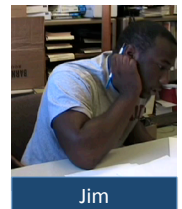
[Before inconsistency is resolved] Would you choose the math answer (pressure greater at 5 meters) or the intuitive one (p greater at 7 meters)?

I would choose the math answer. Perceptions can be misleading. But math gives you the formal answer.

Conclusion

Roadblocks in a student's mathematical reasoning sometimes stem not from deficient mathematical or conceptual knowledge, but from unproductive views about how math connects to other knowledge.

- Curriculum should explicitly attend to these kinds of beliefs



Jim