Improving Engineering Students' Mathematical Sense-making

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Mathematical Sense-making

Looking for coherence and meaning partly by translating between symbolic relations on the page and relations (causal and functional) in the world

- not just manipulating equations, applying algorithms
- central to students' success with modeling and design

Research Objectives

Diagnose student difficulties with mathematical sense-making

- Weak conceptual understanding?
- Inadequate math skills?
- Epistemological issues?
  - Belief that formal and everyday thinking don't connect
  - Belief that equations are just problem-solving tools

Sources of Data: Video of students solving problems alone and in small groups

Determine impact of reformed physics courses on student reasoning in engineering courses

- Curriculum focused on changing students' approaches to learning*
- Follow students from physics course into Basic Circuits Theory and Fluid Mechanics

Sources of Data: Exam answers and scores, survey responses, course grades

Example HW Problem

Standing on a cliff, I take one rock and throw it straight up at a speed of 30 m/s. I take another rock and throw it straight down at 30 m/s. Suppose the cliff is 50 meters high.

(a) ***Just based on common sense***, which rock would be moving faster when it hits the ground, 50 meters below? What's the reasoning for that? → Can't just use equations!

(b) Now find an answer based on the kinematics of constant acceleration: Find x(t) and v(t) for each of the rocks, and find their respective speeds when the hit the ground 50 m below the point of release. → Math skills are still important

(c) Did your answer to (b) agree with your answer to (a)? If not, try to reconcile the contradiction: Figure out what it is about the reasoning in part (a) or part (b) that doesn't work. Get it all to make sense! → Prompting for coherence in math & reasoning

•E. F. Redish & D. Hammer. (Accepted for Publication). Reinventing College Physics for Biologists: Explicating an Epistemological Curriculum, *American Journal of Physics*

** pseudonym used

Project Website: http://umdperg.pbwiki.com/Engineering+Education+Project

Case Study: Making sense of hydrostatic pressure equation

Jim makes a mistake, but sticks to his answer.**

$p = p_{at\ top} + \rho gh$ Is pressure at a depth of 5 meters greater than, less than, or equal to the pressure at 7 meters?

h is negative because it is downward
$p = p_{at\ top} + \rho g(-5)$ vs.
$p = p_{at\ top} + \rho g(-7)$
Pressure at 5 meters is greater.

His mistake does not stem from faulty physical understanding of pressure…

Interviewer

How would a friend who does not know physics answer this?

Someone reasoning with common sense would say pressure is greater at 7 meters, based on experiences under water.

Jim

…or from entrenched difficulties with co-ordinate systems:

Interpreting and using $v = v_0 + at$ for a problem about dropped object

Jim productively chooses "down" as the positive direction

With just a little help he could finally resolve his difficulty.

What about the sign of *g* in that equation?

g should also be negative. …ooooo….the two negatives would cancel. And so pressure at a depth of 7 m *would* be greater.

Although Jim is not *surprised* that math and everyday reasoning "agree"…

[After inconsistency is resolved] What do you think now?

So our common perception *is* correct. Pressure does increase with depth. It makes more sense now.

…his difficulty stems from a *tendency not to try to resolve* inconsistency between formal and everyday knowledge

[Before inconsistency is resolved] Would you choose the math answer (pressure greater at 5 meters) or the intuitive one (*p* greater at 7 meters)?

I would choose the math answer. Perceptions can be misleading. But math gives you the formal answer.

Conclusion

Roadblocks in a student's mathematical reasoning sometimes stem not from deficient mathematical or conceptual knowledge, but from unproductive views about how math *connects* to other knowledge.

- Curriculum should explicitly attend to these kinds of beliefs