Undergraduate Engineers' Sense-making of Math

Mike Hull, Eric Kuo, Ayush Gupta, Andy Elby

EEC-0835880

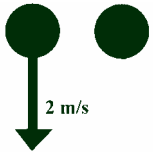
Mathematical Sense-making*

Looking for coherence and meaning partly by translating between symbolic relations on the page and relations (causal and functional) in the world

- not just manipulating equations or applying algorithms
- central to students' success with modeling and design

Case Study: Pat and Alex solve a problem differently

Alex could solve the two balls problem quantitatively



You throw one ball down with speed of 2m/s; You let go of the other. What is the difference in speed of the two balls after 5 seconds – equal to 2?

I plug into the velocity equation. I use the acceleration and the initial velocity, multiply by the time (5 seconds), subtract, and get 2.

…but couldn't make sense of the answer

Interviewer

Could somebody who knows this equation answer without calculating explicitly?

I think you'd have to plug in the numbers because… I mean you just would to be sure. I don't think you can just guess about it.

Alex

Pat found a way to see why the difference SHOULD remain the same

I would think of the equations. I'm trying to think of calculus as well and what the differences *do*. The acceleration is a constant and that means that velocity is linearly related to time and they're both at the same… so the first difference is the same. If the first differences are the same then the difference between the two speeds should not change.

Pat

How can we account for these two different approaches?

Maybe there's a difference in how they think about the equation…

References

*Tuminaro & Redish, Elements of a Cognitive Model of Physics Problem Solving: Epistemic Games. *Physical Review Special Topics: Physics Education Research*, **3**, 2007

Redish & Smith, Looking Beyond Content: Skill Development for Engineers. *Journal of Engineering Education* , July 2008

** Sherin, How Students Understand Physics Equations. *Cognition and Instruction*, 19(4), 2001

Student cartoon figures reproduced from: Allie, Buffler, Kaunda, Campbell, & Lubben, First year physics students' perceptions of the quality of experimental measurements, *International Journal of Science Education*, 20(4), 1998, 447-459

Project Website: http://umdperg.pbwiki.com/Engineering+Education+Project

This work is supported in part by NSF grants EEC-0835880 and REC 04-40113

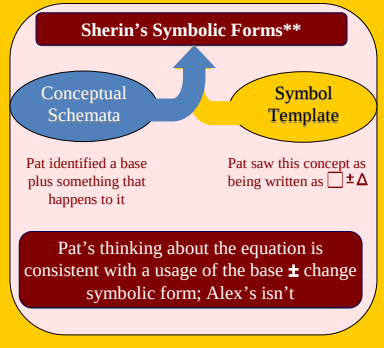
Pat integrated physical intuition with the structure of the equation; Alex didn't

$v=v_0+at$, how would you explain this equation?

Alex focused on what the equation is used for

Alex

I don't really *laugh*… I'm not too sure what I would say. I guess it's interesting because you can find the velocity at any time if you have the initial velocity, the acceleration, time...



Pat connected the terms to what physically happens

Pat

I guess to get the intuition part… you start from an initial velocity and then the acceleration for a certain period of time increases or decreases that velocity… that would be the significance of each term.

We see the same difference in the context of an unfamiliar equation

$p = p_{at\ top} + \rho g h$, how would you explain this equation?

Alex again treated the equation as a plug-and-chug instrument

if you know the density of water and how deep under the water you are, if you multiply those two values… and then… if you add those two together, then you'll know the pressure.

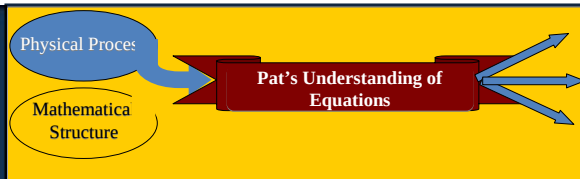
Alex

Pat identified a deeper, conceptual structure

Pat

It reminds me of potential energy problems or any problem where you have a certain condition and then something else happens to it, say a force or energy transfer, that's expressed as a change and an initial and final condition.

Conclusion



- Actively sense-makes while using algebra
- Solves quantitative problem qualitatively
- Also looks for meaning in units and explicitly disparages limiting the meaning of an equation to that of a computational tool (like Alex does)

Understanding equations as being a blend of mathematical structure and narrative of the physical process can enable students to find meaning in solutions to quantitative problems.

Curriculum should attend to fostering this kind of understanding