New Lenses on Developing Physics Expertise

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UDP Expertise

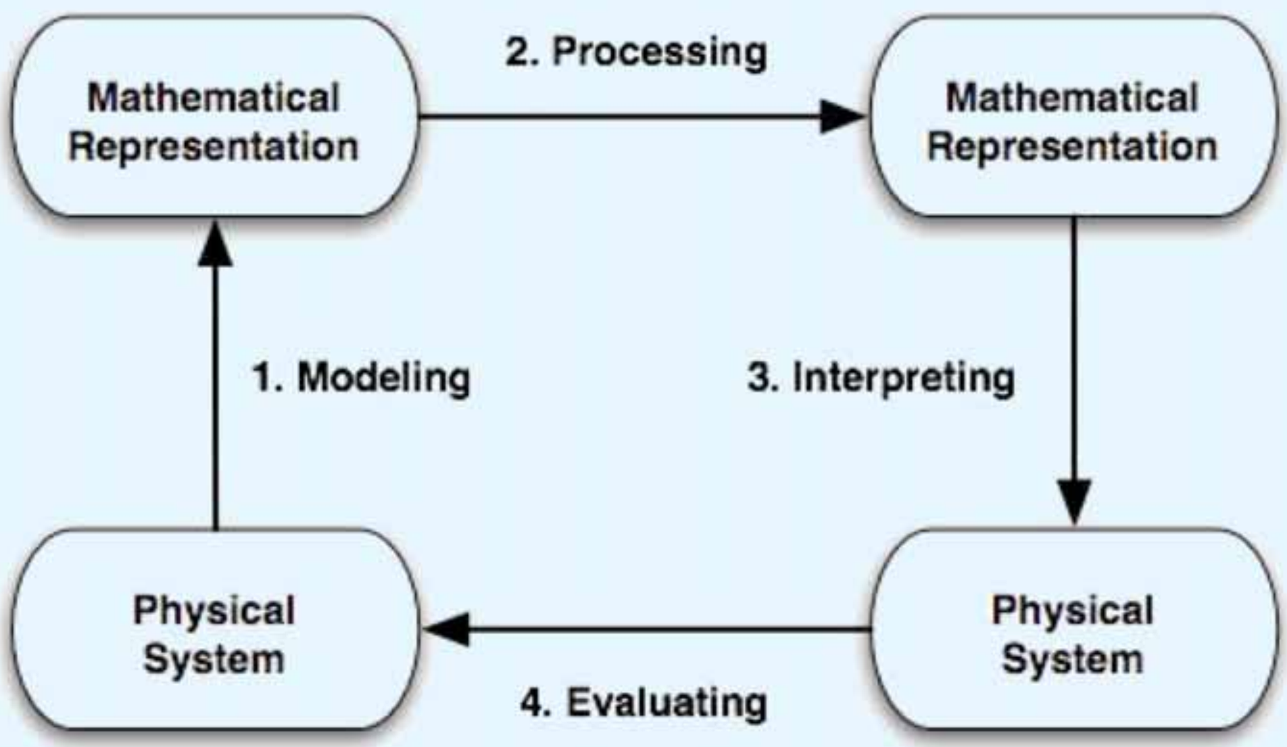
- UDP involves much more math than lower division physics – but the additional factors are not just math.
- What should we be looking for in order to make sense of the transition from novice to expert physicist?

Some lenses

- Modeling
- Blending
- Framing / Epistemology
- Semantics / Making Meaning

Modeling

A Model of Modeling



Note: This is an abstract model of the structure of the relationships between physical and mathematical structures; it is **not** a cognitive model of how people think about it. It is to remind an instructor to consider the aspects of the problem; it is **not** a prescription for how to proceed.

Sample problem

- When we consider the properties of a spring, we typically imagine an ideal spring, which perfectly satisfies Hooke's law, $T = k\,\Delta s$. (I like to read this: if you pull on the opposite ends of a spring with a tension, T, the spring's length will change by an amount Δs, proportional to T with a constant k that is a property of the spring.) In actual fact, this is an awful approximation for most extensions or compressions of the spring. What we typically do is stretch the spring some amount beyond its rest length, say by hanging a weight from it. For small displacements around that equilibrium position, the extra force exerted by the spring is linear around the resting point: "$F = -kx$."
- Consider the spring shown in the figure. Its resting length is 5 cm. Consider that a pair of equal and opposite forces of magnitude, T, are exerted on the two ends of the spring, pulling or pushing in opposite directions. If T is positive, it means the forces are pulling to try to stretch the spring. If T is negative, it means they are trying to compress the spring. Sketch a graph of how the length of the spring varies as a function of T, considering both positive and negative values of T and going to very large values. Make plausible guesses for the values of the length when unusual things happen. Identify the "Hooke's law" regimes and identify salient features of your graph, described what is happening at those salient points.

Sample problem

The pair of coupled non-linear ODEs
are referred to as the Lotka-Volterra
equations and are supposed to represent
the evolution of the populations of
a predator and its prey in time.
The constants A, B, C, D are positive.

$$\frac{dx}{dt} = Ax - Bxy$$

$$\frac{dy}{dt} = -Cy + Dxy$$

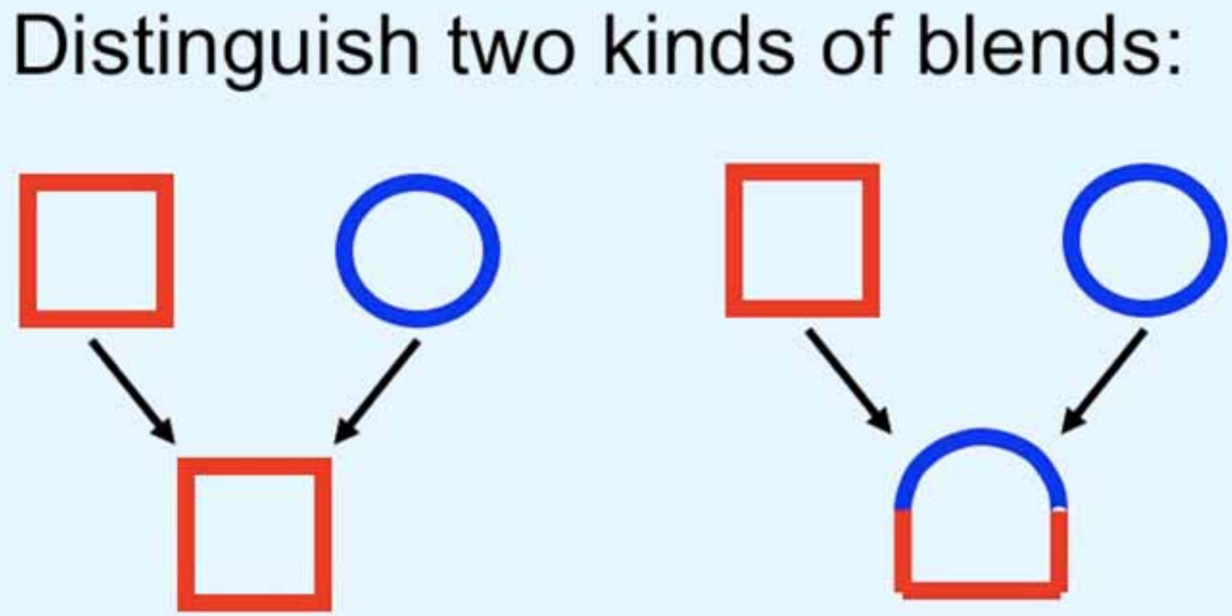
- *Which of the variables, x or y, represents the predator?*
- *Which represents the prey?*
- *What reasons do you have for your choice?*
- *What's left out of this model?*

Blending

Cognitive Blending

- A compression of two or more mental spaces resulting in emergent meaning
  - Mental Space – An organizing frame containing elements and processes relating and transforming those elements
  - Emergent meaning – Relations, inferences, and interpretations that could not occur in either input mental space alone

Types of blends



Non-structural blending:
elements of a second space are imported into the structure and relationships of the first.

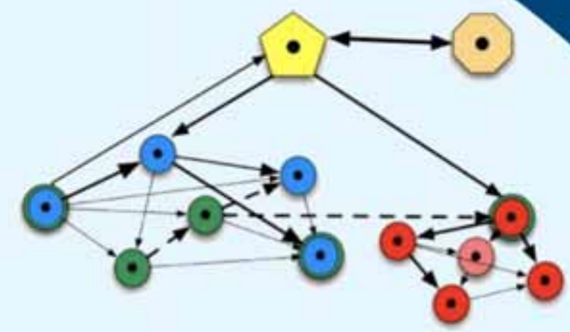
Structural blending:
elements and the structure two spaces are combined, using parts of both to create an emergent space with creative properties present in neither of the original spaces.

Fauconnier, G. and Turner, M. (2002). *The way we think: Conceptual blending and the mind's hidden complexities.* Perseus Books Group.

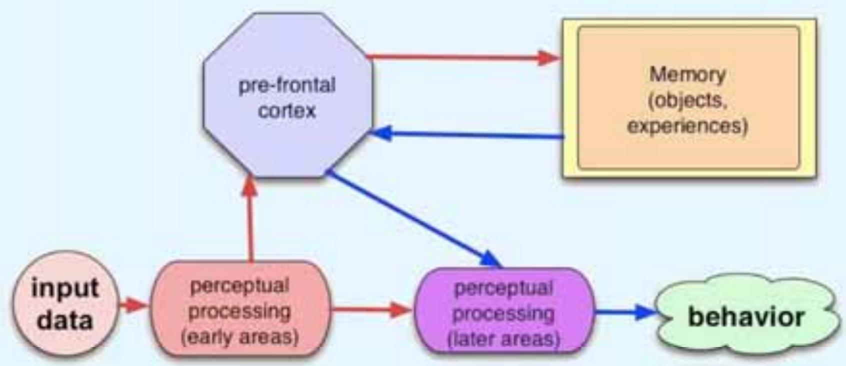
Framing

Framing controls selective attention

- We refer to the process by which bits of perceptual data lead to choices of what data to pay attention to and what knowledge to activate as <u>*framing.*</u>
- If the knowledge being activated is about knowledge and its construction, we call it <u>*epistemological framing.*</u>



Framing is the process that answers the question: “What’s going on here?”



The “concepts” in the choices related to knowledge use

- **Framing** directs selective attention.
- **Epistemological framing** directs attention to the kind tools and evidence you are going to use.
- The general kinds of evidence you choose are **epistemological resources**.
- The specific reason for believing an argument used in a particular example is called a **warrant**.

Examples of e-resources/framing

- *Calculation* – algorithmically following a set of established computational steps should lead to a trustable result.
- *Physical mapping* – a mathematical symbolic representation faithfully characterizes some feature of the physical or geometric system it is intended to represent.
- *Invoking authority* – information that comes from an authoritative source can be trusted.
- *Mathematical consistency* – mathematics and mathematical manipulations have a regularity and reliability and are consist across different situations.

Careful!

- These are NOT intended to describe distinct cognitive structures. Rather, we use them to emphasize different aspects of what may be a unitary non-separable process: the process of judging what knowledge applies in a particular situation.
  - *Framing* – focuses attention on the interaction between cue and response. (You decide you need to find a known theorem.)
  - *Resource* – focuses on the general class of warrant being used. ("You can trust the results in a reliable source such as a textbook.")
  - *Warrant* – focuses on a specific argument, typically using particular elements of the current context. ("Since the path integral of a conservative force is path independent, these two integrals will have the same value.")

The Problem of Grain Size

- If a student primarily attempts to solve a problem largely invoking tools associated with only a single epistemological resource, we then say they are framing the problem epistemologically as “calculation”, or “physical mapping,” etc.
- For this situation, epistemological resources and epistemological framing may appear to be the same thing. But in more general situations, framing may activate multiple resources.

Implications

- Often, when we observe “student difficulties”, what we are seeing are not conceptual difficulties but epistemological ones.
- When a student’s knowledge is not fully coherent (and no one’s knowledge ever is), which bit of one’s knowledge they choose to use in a particular situation can be critical.
- We begin to view student knowledge as a more complex structure with framing and epistemological components – structures that control access to the conceptual elements of a students’ knowledge.

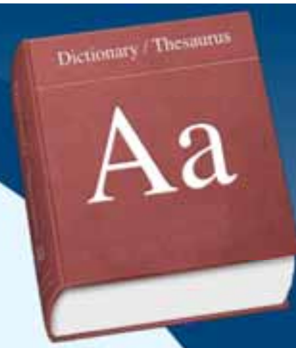
Semantics / Making Meaning

Basic ideas for meaning building

- *Embodied cognition:*
  The meaning of words is grounded in physical experience.
- *Encyclopedic knowledge*:
  Ancillary knowledge is critical in the creation of meaning.
- *Conceptualization*:
  Meaning is constructed dynamically.

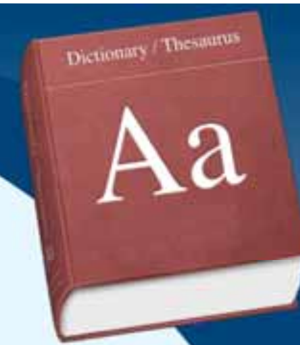
Vyvyan Evans & Melanie Green, *Cognitive Semantics*

Dictionary meaning?



- I do not read arabic, but I know a bit about how an arabic dictionary might work.
  - It would read from right to left.
  - The word to be defined would be on the right of an entry.
- From that, I could find the word "الألكترونات" given enough time.
- But finding it would not help me figure out what it means.
- So how does a dictionary work?

Dictionary meaning!



- Dictionaries are fundamentally circular: words are defined in terms of words.
- The value of a dictionary lies in the hope that as you traverse the circle, you will come upon some set of terms that you already know (and have ultimately learned in some other way than from definitions).

Embodied cognition

- In the end, all of our understanding of even complex concepts must come down to direct perceptual experience.
- Many processes enable the building of this extraordinary and complex linguistic structure:
  - Metaphor (Lakoff & Johnson)
  - Polysemy (Langacker, Evans)
  - Blending (Fauconnier & Turner)

"Encyclopedic" knowledge

- We interpret the words we hear
  or read in terms of a vast knowledge
  and experience of the world.

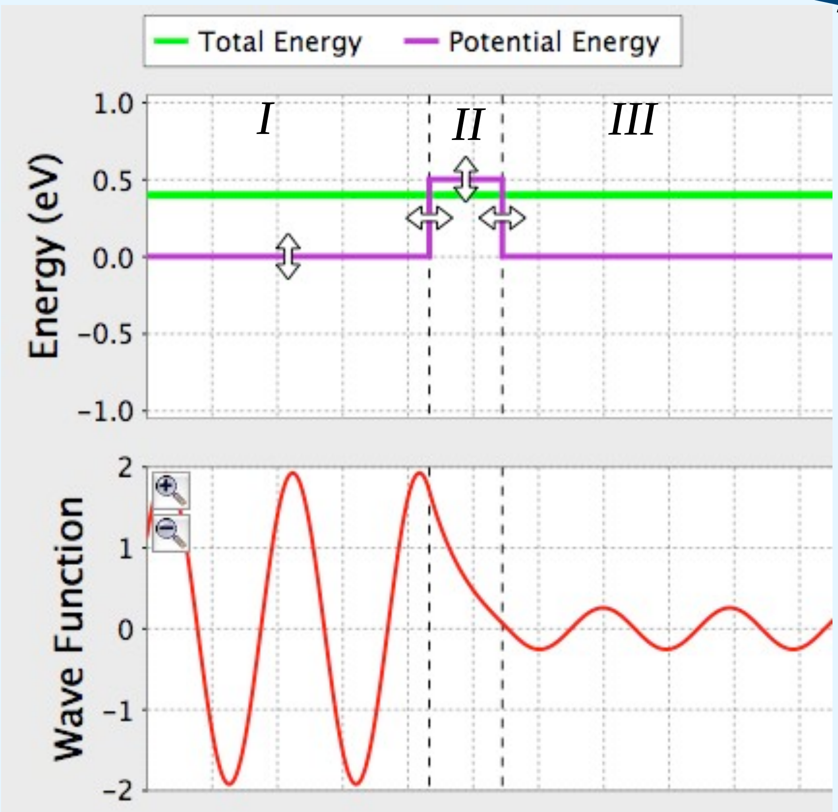
- (I know: Real encyclopedias are
  made up of words just like dictionaries –
  just more of them.
  Maybe this is not a good term.)

Conceptualization

- Language does not directly code for semantic meaning.
- Rather, linguistic units are prompts for the construction of meaning within a given conceptual / contextual frame.
- This means that meaning is dynamically constructed – a process rather than something fixed and stable.

Example: Undergrad QM

- The following problem was given in the second term of UG QM.
  - A beam of electrons of energy E is incident on a square barrier of height V_0 and width a. Find the reflection and transmission coefficients, R and T.
- The student in this example followed an expected procedure but was unable to recover from minor errors.



*

$$\psi_I = Ae^{ikx} + Be^{-ikx}$$

$$\psi_{II} = Ce^{\kappa x} + De^{-\kappa x}$$

$$\psi_{III} = Ee^{ikx} + Fe^{-ikx}$$

$$k^2 = \frac{2mE}{\hbar^2}$$

$$\kappa^2 = \frac{2m(V-E)}{\hbar^2}$$

* CU PhET simulation

$$\psi_I\big|_{x=0} = \psi_{II}\big|_{x=0} \qquad \psi_I'\big|_{x=0} = \psi_{II}'\big|_{x=0}$$

$$\psi_{II}\big|_{x=a} = \psi_{III}\big|_{x=a} \qquad \psi_{II}'\big|_{x=a} = \psi_{III}'\big|_{x=a}$$

4 equations in 6 unknowns

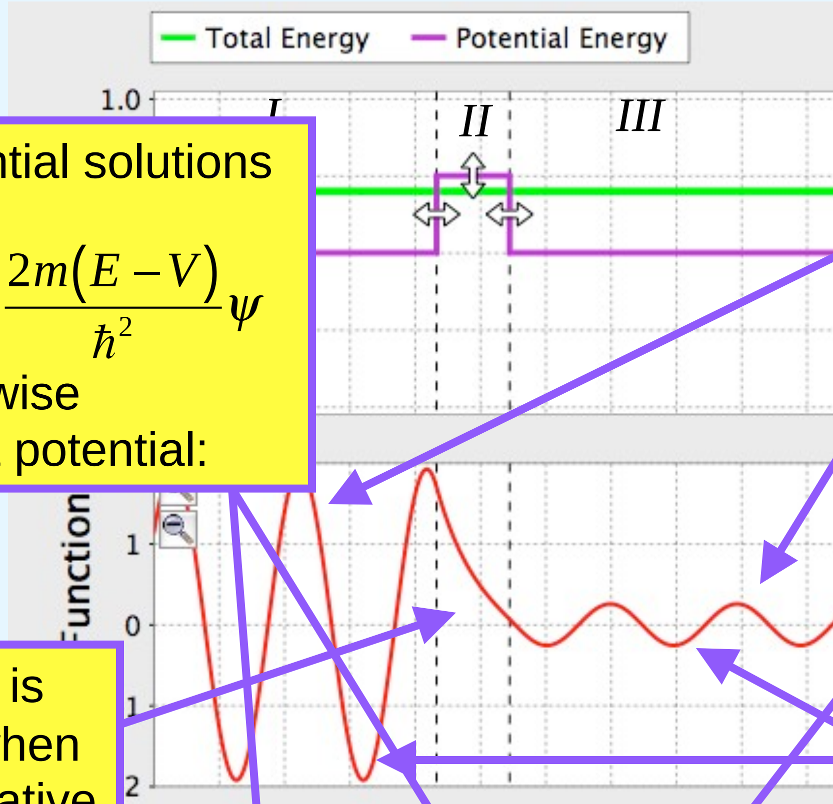
take $F = 0$

churn: solve for B, C, D, E in terms of A.

$$R = \frac{|B|^2}{|A|^2} \qquad T = \frac{|E|^2}{|A|^2}$$

Meaning arising from context

- In lecture, the instructor showed the solutions in each region and the student had copied them down.
- He made some mistakes in copying – keeping the “i’s” in the wave function’s exponents in region II.
- He was totally stuck – kept looking through notes and text trying to find the “correct” form.
- He later showed that he was easily able to generate the solution from the SE.



Exponential solutions from SE

$$\frac{d^2\psi}{dx^2} = -\frac{2m(E-V)}{\hbar^2}\psi$$

in piecewise constant potential:

Wavefunction oscillates when energy is positive

Change of sign of V-E leads to reversal of character of solutions.

Amplitude changes through barrier, but λ stays same (because E same on both sides)

Wavefunction is exponential when energy is negative

$$\psi_I = Ae^{ikx} + Be^{-ikx}$$

$$\psi_{II} = Ce^{\kappa x} + De^{-\kappa x}$$

$$\psi_{III} = Fe^{ikx} + Fe^{-ikx}$$

$$k^2 = \frac{2mE}{\hbar^2}$$

$$\kappa^2 = \frac{2m(V-E)}{\hbar^2}$$

Growing exponential still present since region is bounded (doesn't go to infinity)

* CU PhET simulation

$$\psi_I\big|_{x=0} = \psi_{II}\big|_{x=0} \qquad \psi_I'\big|_{x=0} = \psi_{II}'\big|_{x=0}$$

$$\psi_{II}\big|_{x=a} = \psi_{III}\big|_{x=a} \qquad \psi_{II}'\big|_{x=a} = \psi_{III}'\big|_{x=a}$$

Continuous wavefunction and derivatives correspond to no infinite potentials.

4 equations in 6 unknowns

take $F = 0$

churn: solve for B, C, D, E in terms of A.

$$R = \frac{|B|^2}{|A|^2} \qquad T = \frac{|E|^2}{|A|^2}$$

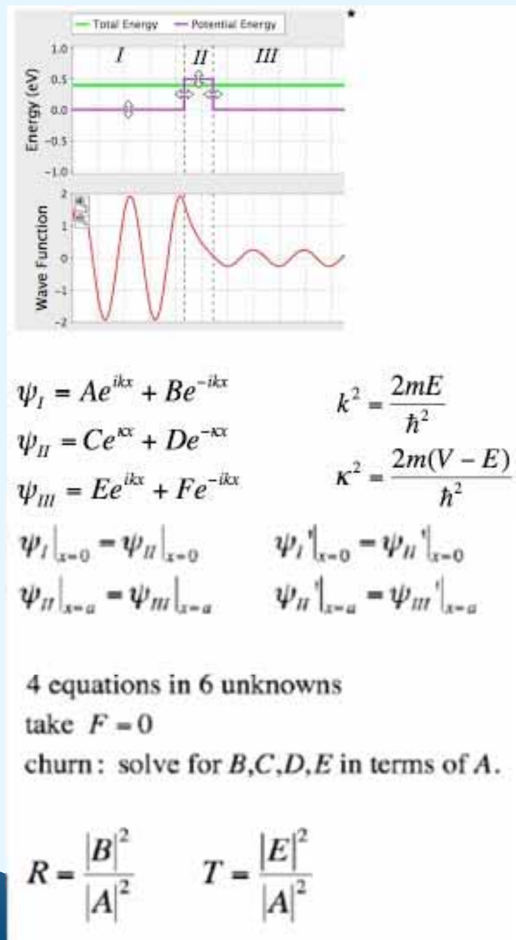
Treatment of solution relies on understanding of meaning of traveling waves.

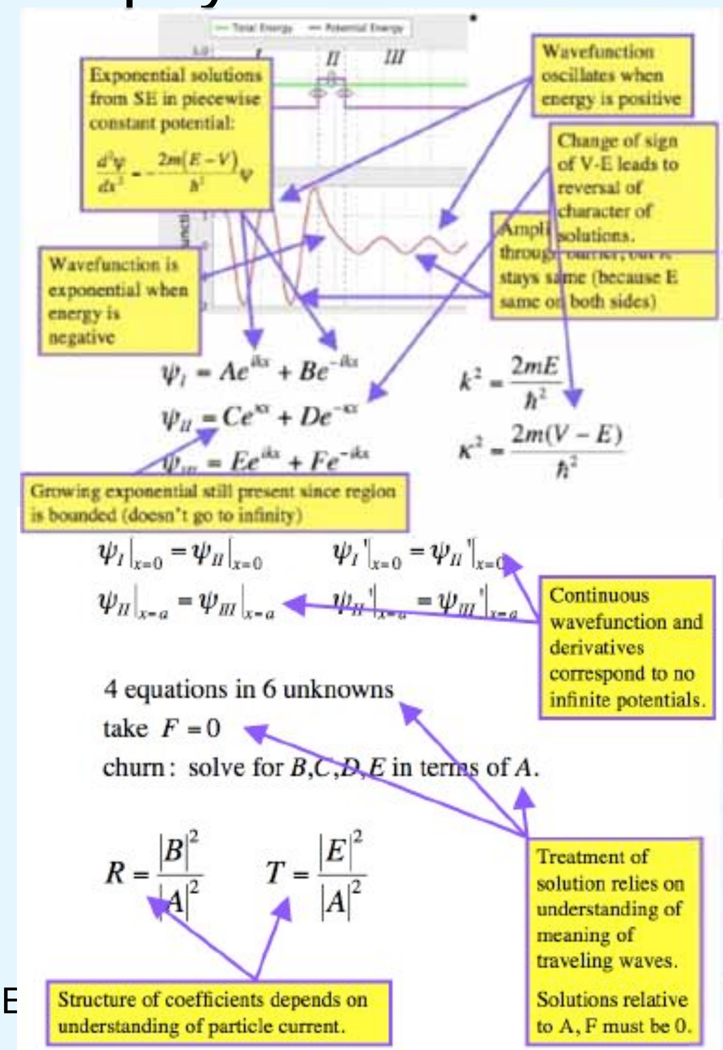
Solutions relative to A, F must be 0.

Structure of coefficients depends on understanding of particle current.

Novice vs. Expert

- The novice solution gets the math
- The expert adds a web of physics associations





Case Study I

- Student returning to complete a physics major after some years in the workplace.
- Took “Intermediate Mathematical Methods” by exam.
- A few weeks after the exam
(before he received his score),
he was interviewed about how he reasoned on some of the exam problems
– including the following.

T. Bing, “An Epistemic Framing Analysis of Upper-Level Physics Students’ Use of Mathematics” PhD Thesis, U of Md (2008) http://www.physics.umd.edu/perg/dissertations/Bing/

An exam problem requiring multiple epistemological resources

- In class, we derived the integral constraint that expressed the conservation of matter of a fluid:

$$-\frac{d}{dt}\int_{\tau} \rho \, d\tau = \int_{\partial\tau} (\rho\vec{v}) \cdot d\vec{A}$$

- Suppose that ρ describes the concentration of a chemical compound in a solution and that compound can be created or destroyed by chemical reactions.
- Suppose also that the rate of creation (or destruction) of the mass of the compound per unit volume as a function of position at the point at a time *t* is given by *Q*(*r*, *t*). *Q* is defined to be positive when the compound is being created, negative when it is being destroyed.
- How would the equation above have to be modified?

This problem is written so as to probe how well a student can integrate physical and mathematical knowledge.

The solution

$\int_\tau \rho \, d\tau$ represents the total mass in the volume

$-\frac{d}{dt}\int_\tau \rho \, d\tau$ represents the rate at which the volume is <u>losing</u> mass

$\int_{\partial\tau} (\rho\vec{v}) \cdot dA$ represents the rate at which mass is flowing out of the volume

$Q(\vec{r},t)$ represents the rate mass is created at a point (a density)

$Q(\vec{r},t)\,d\tau$ represents the rate mass is created in a small volume, *dτ*

$\int_\tau Q(\vec{r},t)\,d\tau$ represents the rate at which mass is created in the volume, *τ*

Therefore, the equation must look like: $\pm\int_\tau Q(\vec{r},t)\,d\tau - \frac{d}{dt}\int_\tau \rho \, d\tau = \int_{\partial\tau} (\rho\vec{v}) \cdot dA$

We choose the sign by considering a particular physical situation (e.g., *Q* positive so stuff is created inside, but it all flows out so the total inside stays the same)

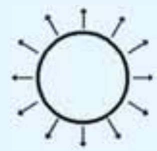
$$\int_\tau Q(\vec{r},t)\,d\tau - \frac{d}{dt}\int_\tau \rho \, d\tau = \int_{\partial\tau} (\rho\vec{v}) \cdot dA$$

Figuring out the sign

- "…yeah the one thing I was confused about on the exam and I continue to be confused about it now, is the sign of this here, [*writes "+/-" in front of Q*] like whether this is going to be a plus or a minus because, rate of creation, so if it's getting created, and then it's - Yeah, I'm not sure about this one, about this sign."

$$\pm Q(\vec{r},t) - \frac{d}{dt}\int_{\tau} \rho\, d\tau = \int_{\partial\tau} \left(\rho \vec{v}\right) \cdot dA$$

After some prodding

- “Uhhh, yeah, if it’s a, if it’s a positive sign then the right hand side has to increase [*points to* $\int_{\partial\tau}(\rho\vec{v})\cdot dA$] because something is getting sourced inside this volume. So for this to increase- [*points to picture:* 

  Yeah, so it can not be a positive, it has to be a negative, because then that’s going to increase - for these signs to match, for the magnitude to increase like these signs have to match, [*Erases “±” and writes “-Q”*]
  so it’s probably negative. Although on the other hand, when I think of a source I think of a positive sign and sink is a negative sign. Yeah so that’s where my confusion lies.“

A lot of mistakes, but…

- This student made some serious errors
  - didn't check units and failed to identify Q as a density ("per unit volume").
  - misapplied his physical reasoning and got the wrong sign.
- But the student exhibits an epistemological framing that values coherency among multiple lines of reasoning.
- He explicitly uses *Physical Mapping*, *Calculation*, and *Invoking Authority* as interacting sub-frames nested within a larger coherency-valuing epistemological framing

Meaning in math?

- These ideas suggest that meaning relies on
  - Organized knowledge structures (frames)
  - Associational patterns and activation (framing)
  - The linking of different kinds of knowledge (blending)
- Applying it to math-in-physics they suggest that meaning derives from the association of math with a physical context and physical ideas.