Characterizing expertise in physics problem solving

Edward F. Redish
University of Maryland

Tom Bing
Emory University

Work supported in part
by NSF grant DUE 05-24987.

AAPT Portland, OR
7/20/10

Learning Physics

There has been a lot of education research literature exploring Novice – Expert differences.

Novice

Journeyman

Expert

But if we really want to understand how one develops into an expert physicist, we need to look at intermediate stages.

The Problem

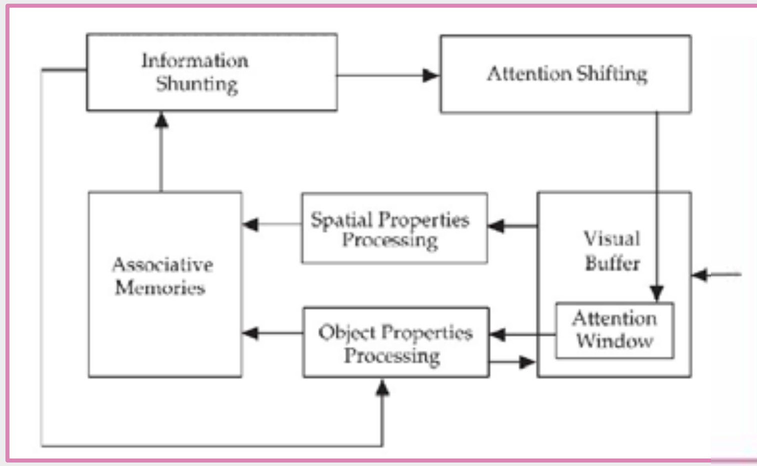
- When learning to become an expert in a complex subject such as physics, you learn a huge amount of stuff.
- The brain can only manipulate a small amount of stuff at any given time. (The problem of working memory.)
- Any given problem may require a subset (and perhaps a large one) of your knowledge.
  - ***How do you learn to manipulate a larger set of information than your working memory can hold?***
  - ***How do you select the knowledge you need out of all the things you know?***

The beginnings of an answer

- *How do you learn to manipulate more information than your working memory can hold?*
  - By chunking: creating abstract objects with complex properties that can be "unpacked" as needed.
  - By externalizing: writing down equations that can be quickly referenced allowing shifts of your mental processing.
- *How do you select the knowledge you need out of all the things you know?*
  - By blending: creating compact mental structures that integrate physical and mathematical knowledge.
  - By framing: selecting a limited mental "search space" to restrict the knowledge to bring to bear.

Focus on this today.

Psychology & Neuroscience models increasingly include decision-making ("Choice") feedback loops



Kosslyn (2007)

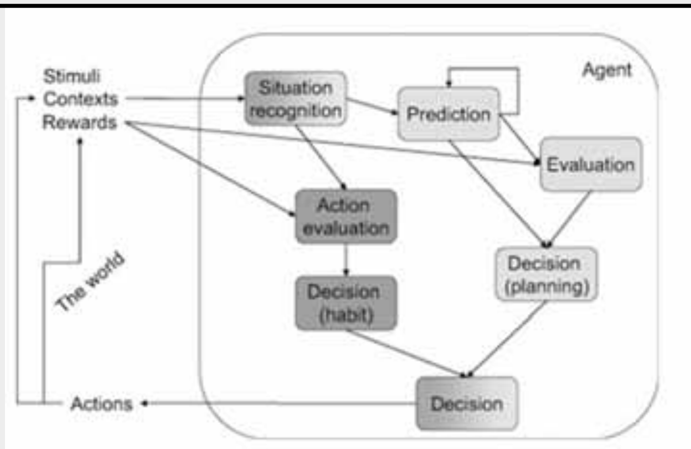


Figure 1. Structure of decision making in mammalian agents. Components of the more flexible, planning-based system are shown in light gray; components of the less flexible, habit-based system are shown in dark gray. Components involved in both are shown in gradient color.

Redish, Jensen, & Johnson (2008)

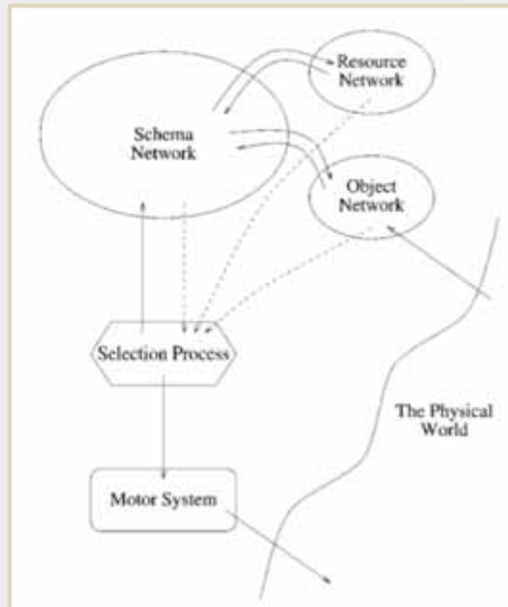


Fig. 1. *The principal components of the contention scheduling implementation.*

Cooper & Shallice (2009)

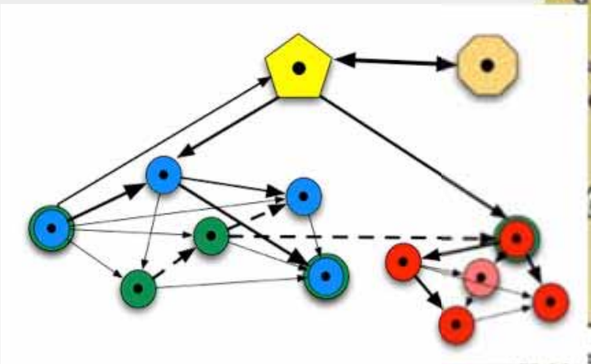
Phemonenological Implications

- Knowledge activation is context dependent.
- Context dependence may be a conscious choice.
- Context dependence may be an unconscious choice
- Student interpretation of the nature of their task may limit what parts of their knowledge they call on.

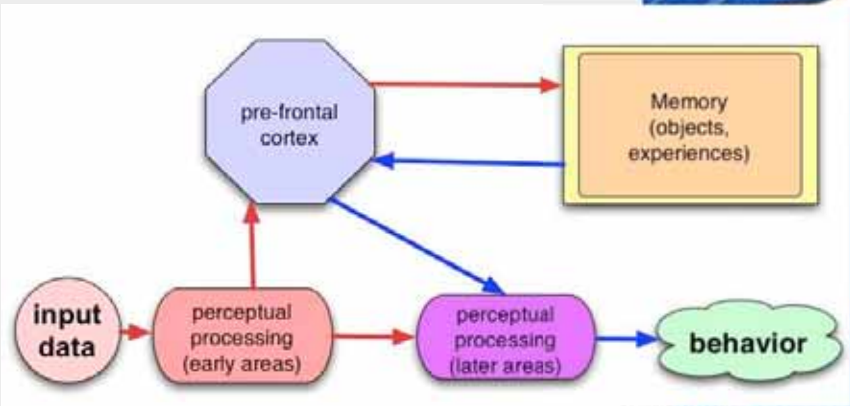
- **Result:**
  In modeling student knowledge structures, we have to pay attention to how they perceive and interpret the nature of the task they are engaged in.

Framing

- We refer to the process by which bits of perceptual data lead to choices of what data to pay attention to and what knowledge to activate as *framing.*
- If the knowledge being activated is about knowledge and its construction, we call it *epistemological framing.*



Framing is the process that answers the question: "What's going on here?"



The "concepts" in the choices related to knowledge use

- **Framing** directs selective attention.
- **Epistemological framing** directs attention to the kind tools and evidence you are going to use.
- The general kinds of evidence you choose are **epistemological resources**.
- The specific reason for believing an argument used in a particular example is called a **warrant**.

Examples of e-resources/framing

- *Calculation* – algorithmically following a set of established computational steps should lead to a trustable result.
- *Physical mapping* – a mathematical symbolic representation faithfully characterizes some feature of the physical or geometric system it is intended to represent.
- *Invoking authority* – information that comes from an authoritative source can be trusted.
- *Mathematical consistency* – mathematics and mathematical manipulations have a regularity and reliability and are consist across different situations.

Careful!

- These are NOT intended to describe distinct cognitive structures. Rather, we use them to emphasize different aspects of what may be a unitary non-separable process: the process of judging what knowledge applies in a particular situation.
  - *Framing* – focuses attention on the interaction between cue and response. (You decide you need to find a known theorem.)
  - *Resource* – focuses on the general class of warrant being used. ("You can trust the results in a reliable source such as a textbook.")
  - *Warrant* – focuses on a specific argument, typically using particular elements of the current context. ("Since the path integral of a conservative force is path independent, these two integrals will have the same value.")

The Problem of Grain Size

- If a student primarily attempts to solve a problem largely invoking tools associated with only a single epistemological resource, we then say they are framing the problem epistemologically as “calculation”, or “physical mapping,” etc.
- For this situation, epistemological resources and epistemological framing may appear to be the same thing. But in more general situations, framing may activate multiple resources.

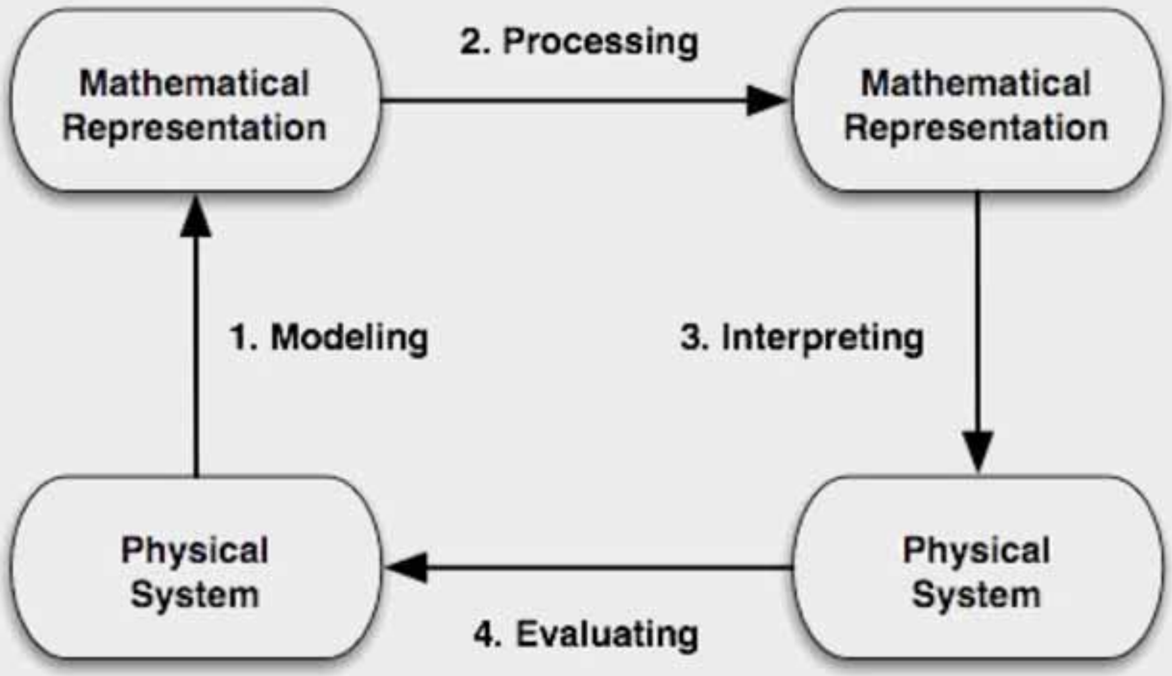
Implications

- Often, when we observe “student difficulties”, what we are seeing are not conceptual difficulties but epistemological ones.
- When a student’s knowledge is not fully coherent (and no one’s knowledge ever is), which bit of one’s knowledge they choose to use in a particular situation can be critical.
- We begin to view student knowledge as a more complex structure with framing and epistemological components – structures that control access to the conceptual elements of a students’ knowledge.

What knowledge is relevant to a journeyman physics student?

- As our physics majors progress to more advanced levels (upper division, e.g.) we include more sophisticated math.
- But sometimes our students don’t realize how much physical knowledge we are integrating into our use of advanced math.
- And sometimes we don’t notice it either!

A Model of Modeling with Math



Note: This is an abstract model of the structure of the relationships between physical and mathematical structures; it is **not** a cognitive model of how people think about it. It is to remind an instructor to consider the aspects of the problem; it is **not** a prescription for how to proceed.

Some implications

- We assume that students learn to think about mechanism and blend the physics and the math a bit at a time, as we did. (Is that the way we did it?)
  - But this may be a filter rather than a normal occurrence.
- We often don't stress the mixing of different knowledge spaces and epistemological resource in our instruction.
  - Even if we do, students may not hear it.
  - Even if we do, we may not have good ways to evaluate it.

Case Study I

- Student returning to complete a physics major after some years in the workplace.
- Took “Intermediate Mathematical Methods” by exam.
- A few weeks after the exam (before he received his score), he was interviewed about how he reasoned on some of the exam problems – including the following.

T. Bing, “An Epistemic Framing Analysis of Upper-Level Physics Students’ Use of Mathematics” PhD Thesis, U of Md (2008) http://www.physics.umd.edu/perg/dissertations/Bing/

An exam problem requiring multiple epistemological resources

- In class, we derived the integral constraint that expressed the conservation of matter of a fluid:

$$-\frac{d}{dt}\int_{\tau}\rho\, d\tau = \int_{\partial\tau}(\rho\vec{v})\cdot d\vec{A}$$

- Suppose that *ρ* describes the concentration of a chemical compound in a solution and that compound can be created or destroyed by chemical reactions.
- Suppose also that the rate of creation (or destruction) of the mass of the compound per unit volume as a function of position at the point at a time *t* is given by *Q*(*r*, *t*). *Q* is defined to be positive when the compound is being created, negative when it is being destroyed.
- How would the equation above have to be modified?

This problem is written so as to probe how well a student can integrate physical and mathematical knowledge.

The solution

$\int_{\tau} \rho \, d\tau$ represents the total mass in the volume

$-\frac{d}{dt}\int_{\tau} \rho \, d\tau$ represents the rate at which the volume is <u>losing</u> mass

$\int_{\partial\tau} (\rho \vec{v}) \cdot dA$ represents the rate at which mass is flowing out of the volume

$Q(\vec{r},t)$ represents the rate mass is created at a point (a density)

$Q(\vec{r},t)\,d\tau$ represents the rate mass is created in a small volume, *dτ*

$\int_{\tau} Q(\vec{r},t)\,d\tau$ represents the rate at which mass is created in the volume, *τ*

Therefore, the equation must look like: $\pm\int_{\tau} Q(\vec{r},t)\,d\tau - \frac{d}{dt}\int_{\tau} \rho \, d\tau = \int_{\partial\tau} (\rho \vec{v}) \cdot dA$

We choose the sign by considering a particular physical situation (e.g., *Q* positive so stuff is created inside, but it all flows out so the total inside stays the same)

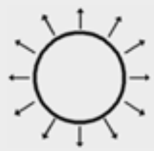
$$\int_{\tau} Q(\vec{r},t)\,d\tau - \frac{d}{dt}\int_{\tau} \rho \, d\tau = \int_{\partial\tau} (\rho \vec{v}) \cdot dA$$

Figuring out the sign

- "…yeah the one thing I was confused about on the exam and I continue to be confused about it now, is the sign of this here, [*writes "+/-" in front of Q*] like whether this is going to be a plus or a minus because, rate of creation, so if it's getting created, and then it's - Yeah, I'm not sure about this one, about this sign."

$$\pm Q(\vec{r},t) - \frac{d}{dt}\int_{\tau} \rho \, d\tau = \int_{\partial\tau} \left(\rho\vec{v}\right)\cdot dA$$

After some prodding

- "Uhhh, yeah, if it's a, if it's a positive sign then the right hand side has to increase [*points to* $\int_{\partial\tau}(\rho\vec{v})\cdot dA$] because something is getting sourced inside this volume. So for this to increase- [*points to picture:* 

  Yeah, so it can not be a positive, it has to be a negative, because then that's going to increase - for these signs to match, for the magnitude to increase like these signs have to match, [*Erases "±" and writes "-Q"*]
  so it's probably negative. Although on the other hand, when I think of a source I think of a positive sign and sink is a negative sign. Yeah so that's where my confusion lies."

A lot of mistakes, but…

- This student made some serious errors
  - didn't check units and failed to identify Q as a density ("per unit volume").
  - misapplied his physical reasoning and got the wrong sign.
- But the student exhibits an epistemological framing that values coherency among multiple lines of reasoning.
- He explicitly uses *Physical Mapping*, *Calculation*, and *Invoking Authority* as interacting sub-frames nested within a larger coherency-valuing epistemological framing

Case Study II

- Situation: Student working with a group to do homework in an UG QM class.
- The task requires evaluating the expectation of $\int x^2 |\psi_n(x)|^2 dx$ in an infinite square well potential.
- The student writes the integral $\frac{2}{L}\int_{-\infty}^{\infty} x^2 \sin^2\left(\frac{n\pi x}{L}\right) dx$ and proceeds to spend 15 minutes demonstrating to her satisfaction that the integral does not converge.

T. Bing & E. F. Redish, "Symbolic manipulators affect mathematical mindsets," Am. J. Phys. **76**, 418-424 (2008) http://www.physics.umd.edu/perg/papers/redish/Bing&Redish.pdf

No mistakes, but…

- The student's calculations were careful, accurate, and mathematically sophisticated [and made good use of external tools, including Mathematica, her powerful calculator, and pencil-and-paper calculation].
- However, her e-framing was restricted, being limited to mathematical computation.
- By failing to activate "Physical Mapping," she missed the fact that she was doing the wrong integral.

A double dissociation

- The first student made a lot of errors, both computational and physical, but strongly attempted to draw on a blend of epistemological resources – physical and mathematical.
- The second student was mathematically precise, but failed to notice that she was doing the wrong integral because of a failure to frame the task with enough epistemological sophistication.

Conclusion

- As our students progress to more complex levels of physics learning, we should begin paying attention not only to the knowledge and math skills they have developed, but also to the issue of how the access, and blend that knowledge.
- The language of epistemological framing / resources/ and warrants may help us better describe this aspect of their learning.