Drawing physical insight from mathematics via epistemic games

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In many areas of science math modeling is not part of the instructional tradition. In the life sciences, efforts to modify instruction to include more math have met with only moderate success. Many life science students resist using math in science, despite their success in math classes. In this project we explore previously unarticulated differences between the way math is used in math and science classes and the reasons behind students' difficulties. Success in the project will provide a better understanding of what needs to be done to help a larger fraction of life science students learn to use math productively. The project's primary goal is to identify students' barriers to using math in science through a mix of quantitative and qualitative research. We will weave an instructional thread on mathematical modeling: readings, clicker questions, homework and exam questions to build students' modeling skills in a variety of contexts. Our modeling thread will be implemented in an Introductory Physics for Life Sciences (IPLS) course. Our early results identify many places where students struggle and provide guidance for creating interviews; we have completed more than 60 so far. These, together with our "Mathematics in Science" instructional thread, should provide useful guidance to other instructional development groups attempting to improve students' integration of math into their scientific toolbox.

Finding meaning in mathematics

The expectations for what an equation means are different in physics and math contexts. [1]
Once you find an equation, how do you know if it is correct?
How do you use an equation to learn something new about what a physical system will do?
How can your physical intuition guide the process of finding equations?

Concept inventory for epistemic games

When we create knowledge by working from a starting point to an end point making moves according to certain rules, we're playing an epistemic game.[2] We are developing the Math Epistemic Games Survey (MEGS) to study how students learn to play epistemic games and use them productively throughout a semester of physics.

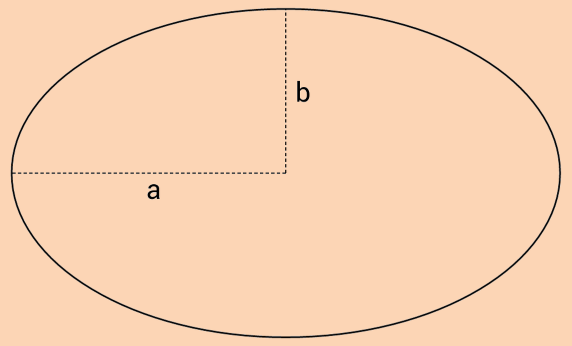
Dimensional Analysis

Which expression could represent the surface area of a solid object? Variables A, B, and C represent lengths, such as the length of the side of an object or the diameter of a circular object.

1. $4\sqrt{ABC(A+B+C)}$
2. $A^2+B^2+C^2$
3. $\frac{\sqrt{2}A^2}{2B}$
4. $\frac{3AC}{B}$
5. None of these could be a surface area
6. I don't know

Examining Extreme Cases

Which of these is the formula for the area of an ellipse?



1. πa^2
2. πb^2
3. $\pi(a+b)$
4. πab
5. $\pi(\frac{a+b}{2})^2$
6. I don't know

Estimation

How many breaths does an average person take in their lifetime?

1. one thousand
2. one million
3. one billion
4. one trillion
5. I don't know

Mapping Symbols to Meaning

You step on a moving sidewalk moving forward at speed *s*. After going a little way, you realize you dropped your wallet before stepping on, so you turn around and run back to the beginning of the sidewalk. Your running speed is *r*. How fast would an observer standing on the ground next to the sidewalk see you moving?

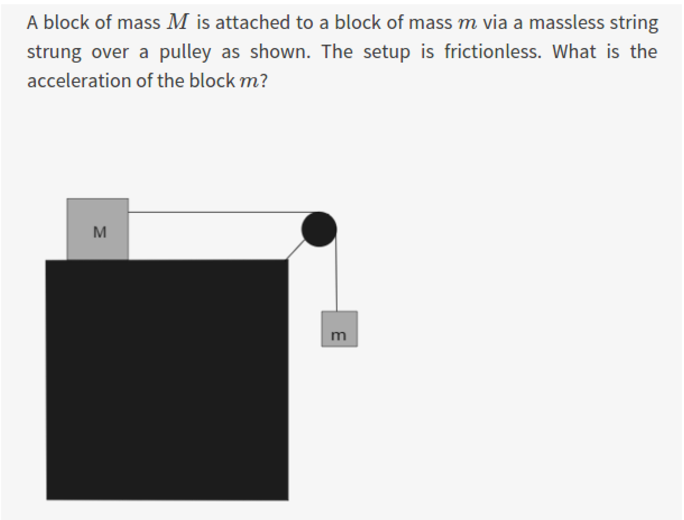
1. r+s
2. r-s
3. r*s
4. r/s
5. I don't know

How do students play epistemic games?

How do students choose what epistemic game to play when solving a problem?
What procedural resources to students use to play each epistemic game?
Are epistemic games viewed as ways of confirming results, or as generating new knowledge?
How does playing an epistemic game affect student expectations surrounding sense-making?
How do students apply epistemic games learned in one context (e.g. geometry) to another (mechanics)?

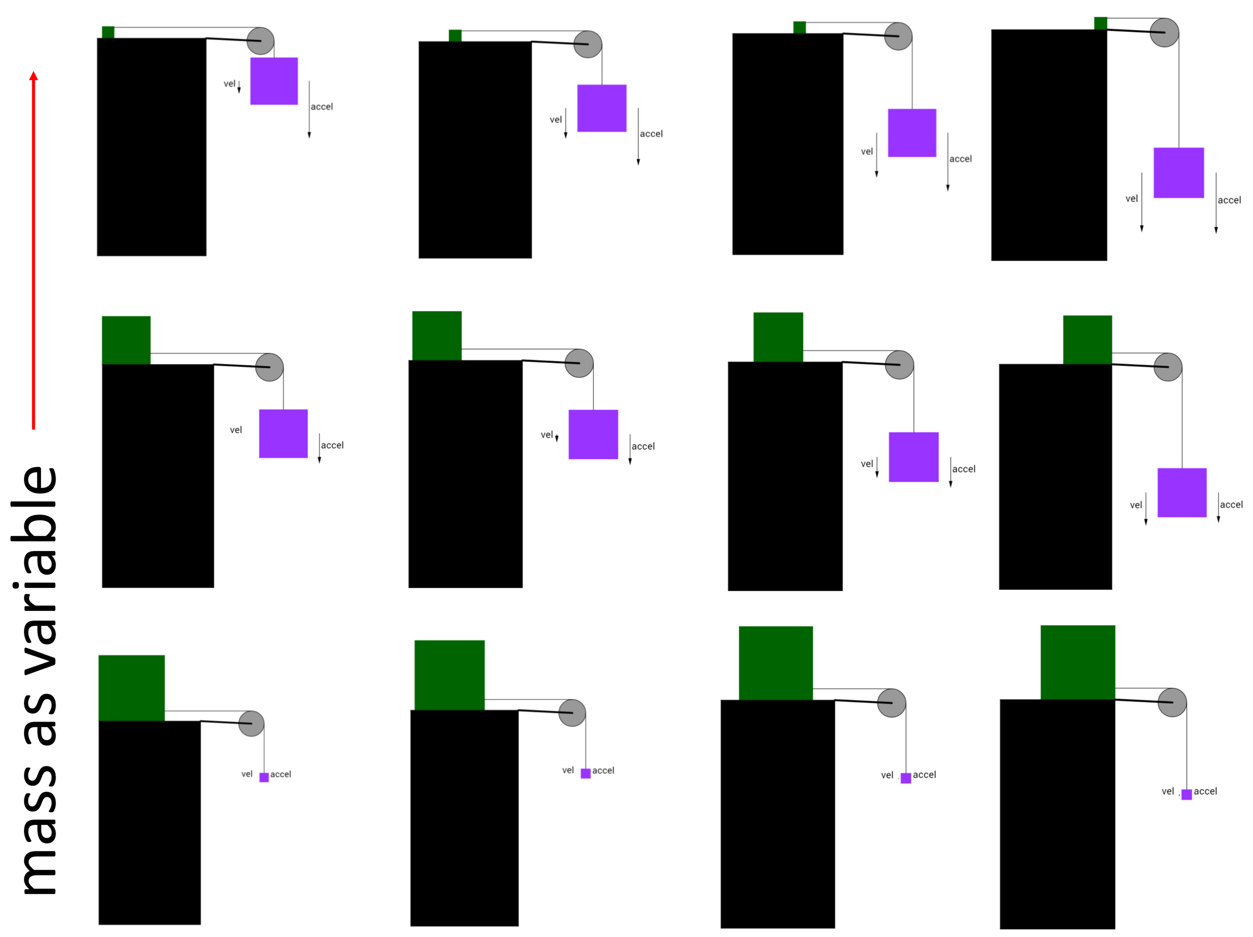
To investigate these questions, we've conducted several rounds of group and individual problem-solving interviews.

Playing "check extreme cases" with the half-Atwood's machine

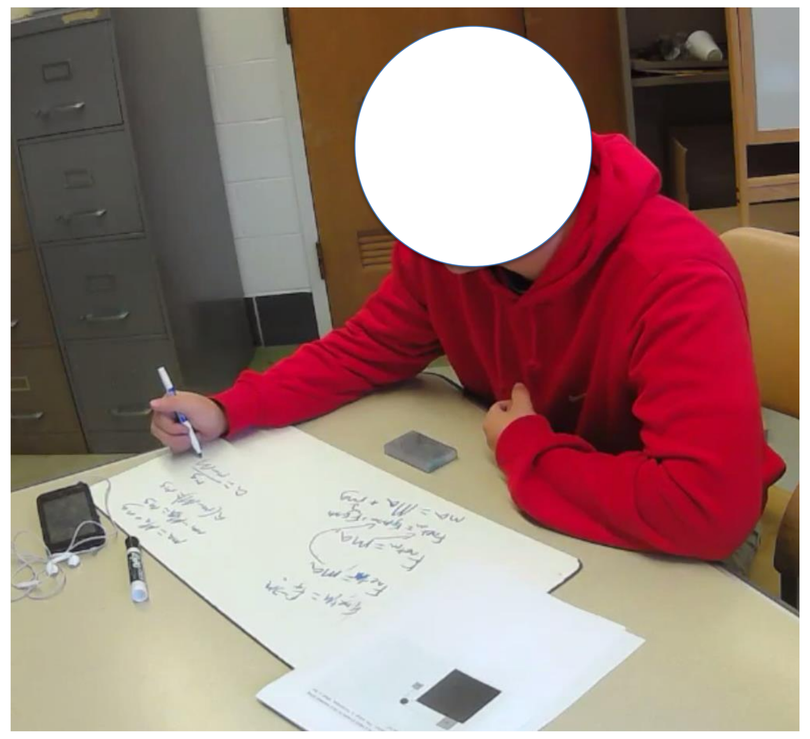


- Intuitions were much more accurate in the extreme cases than in the general case.
- Students analyzing extreme cases of their formulas for acceleration usually only matched the analysis to their physical intuition after being prompted.
- After being prompted to use extreme cases to analyze a physics scenario, students used it spontaneously to solve a geometry problem.
- Analysis of students discussing extreme cases led to interest in the ontology of parameters.
- Extreme cases were mostly useful as evaluative tools, but sometimes placed constraints on equations whose correct forms were unknown

physical motion, stroboscopic changes in mass

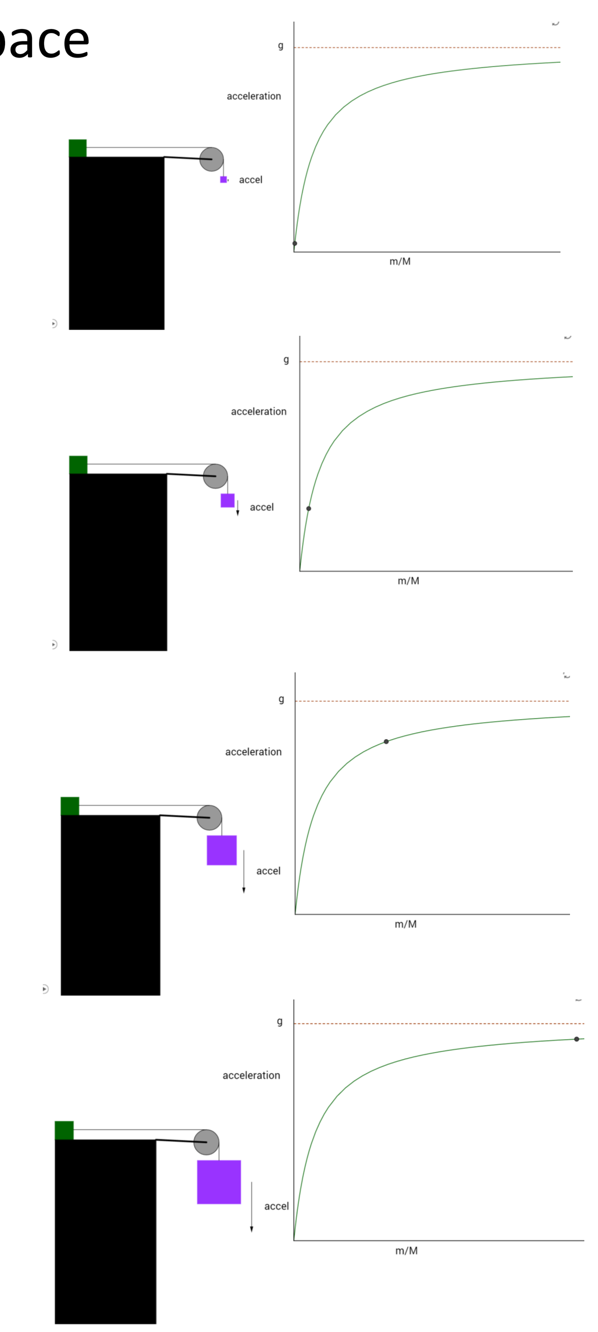


"This is big and this is small and this positive." [draws arrows to m, M and entire expression mg/(M-m)]

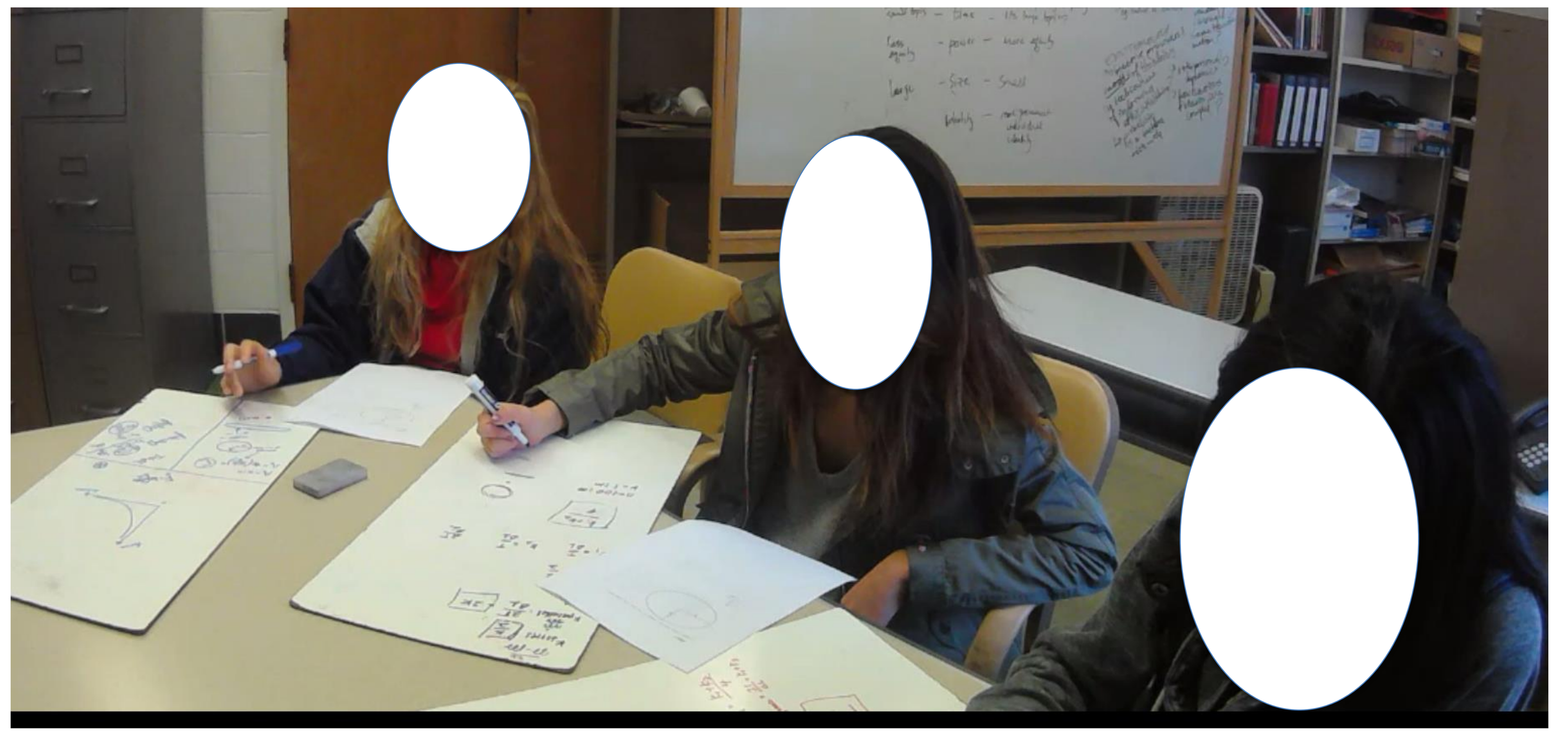


"That's not what should happen. Should be as this one grows [points to M] it [acceleration] gets smaller, so like that [M] has to be in the denominator."

motion occurring in abstract space



Playing "check extreme cases" for the area of an ellipse



"In this case, then, it's almost like a rectangle. I mean, you know the area of a rectangle is just length times width, so that would be a times b."

"I think if the ellipse, if the two radii were like closer equivalent then three and five wouldn't have that big of a difference. We're like looking at extreme cases, then you see."

Checking extreme cases prompted bringing in new analogical resources. Student conversation focused on the disparity between different viewpoints, referred back and forth between pictures and equations freely, and ended in a coherent resolution.

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References
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[2] Tuminaro, Jonathan, and Edward F. Redish. "Elements of a cognitive model of physics problem solving: Epistemic games." *Phys Rev Special Topics-Physics Education Research* 3.2 (2007): 020101.

Acknowledgements
This work was supported by the NSF grants DUE 15-04366, DUE 11-22818 and DRL 05-29482, and the HHMI NEXUS grant. Thanks to the rest of the NEXUS/Physics research team and the UMD Physics Education Research Group.