Rethinking physics for life-science students:
Teaching using math in physics

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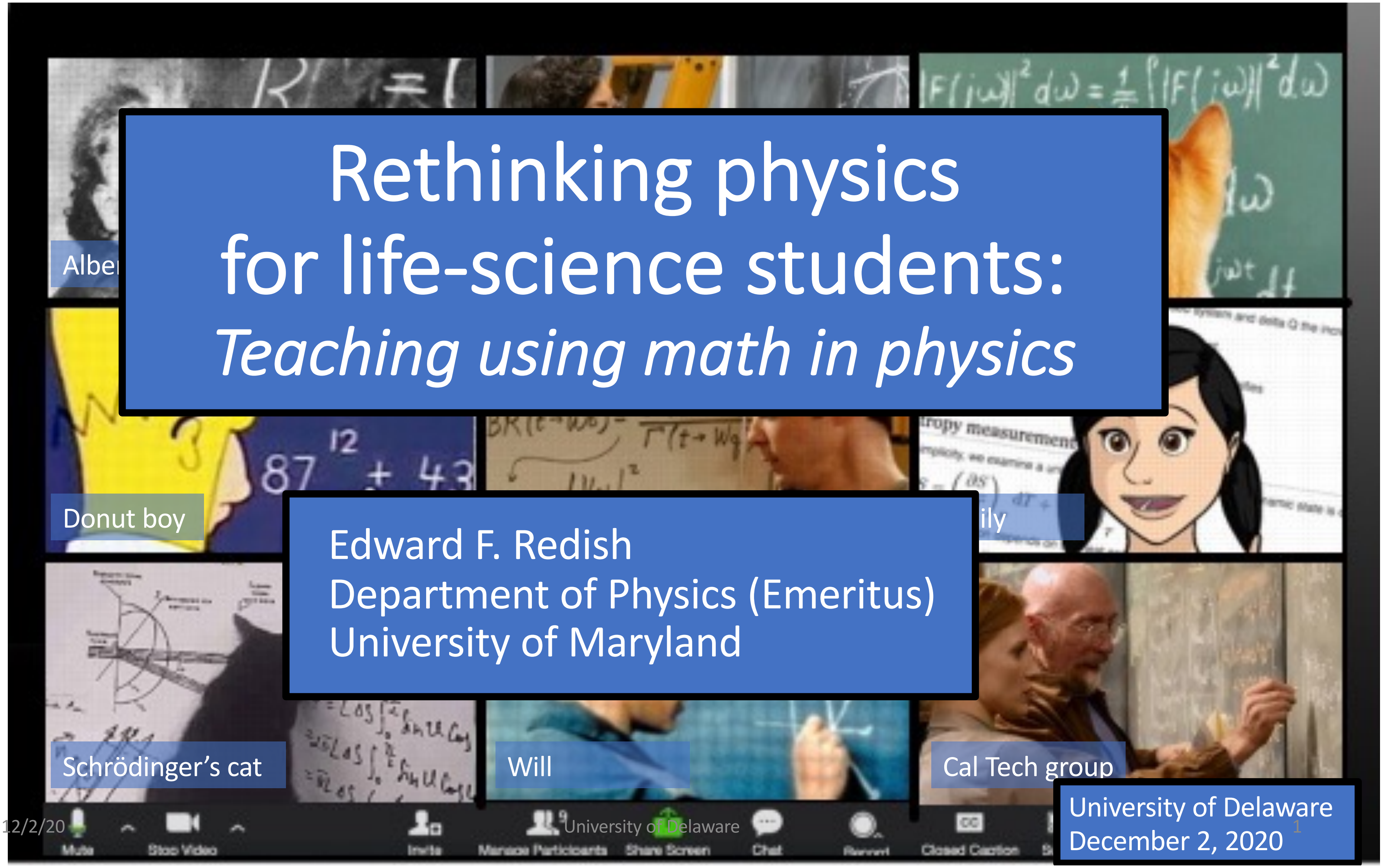

University of Delaware
December 2, 2020

Overview

- In this talk I will discuss what I and my NEXUS/Physics team learned from extensive research and development in an Introductory Physics for Life Science Students class.
- While this was a specific case, the Physics Education Research we did resulted in insights and methods that should be applicable across different populations and at different parts of the curriculum.

Many of the ideas expressed here were developed as part of the NEXUS/Physics project and the Under/Over projects supported by NSF and HHMI. Many people were involved. Major contributors include:

The NEXUS/Physics "Gang of 5"

NΦ NEXUS/Physics
Physics for life-science students

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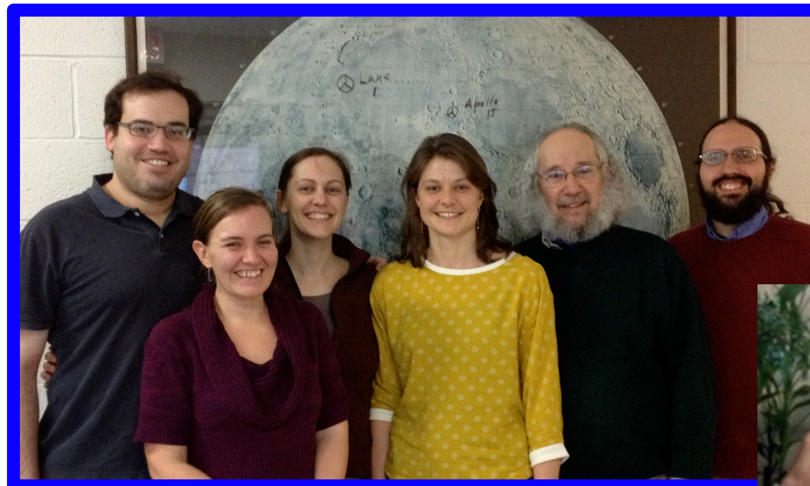

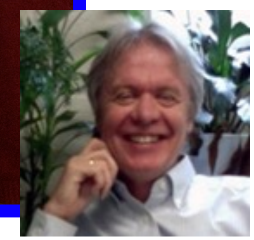

The "Understanding & overcoming barriers to using math in science" project team

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Deborah Hemingway
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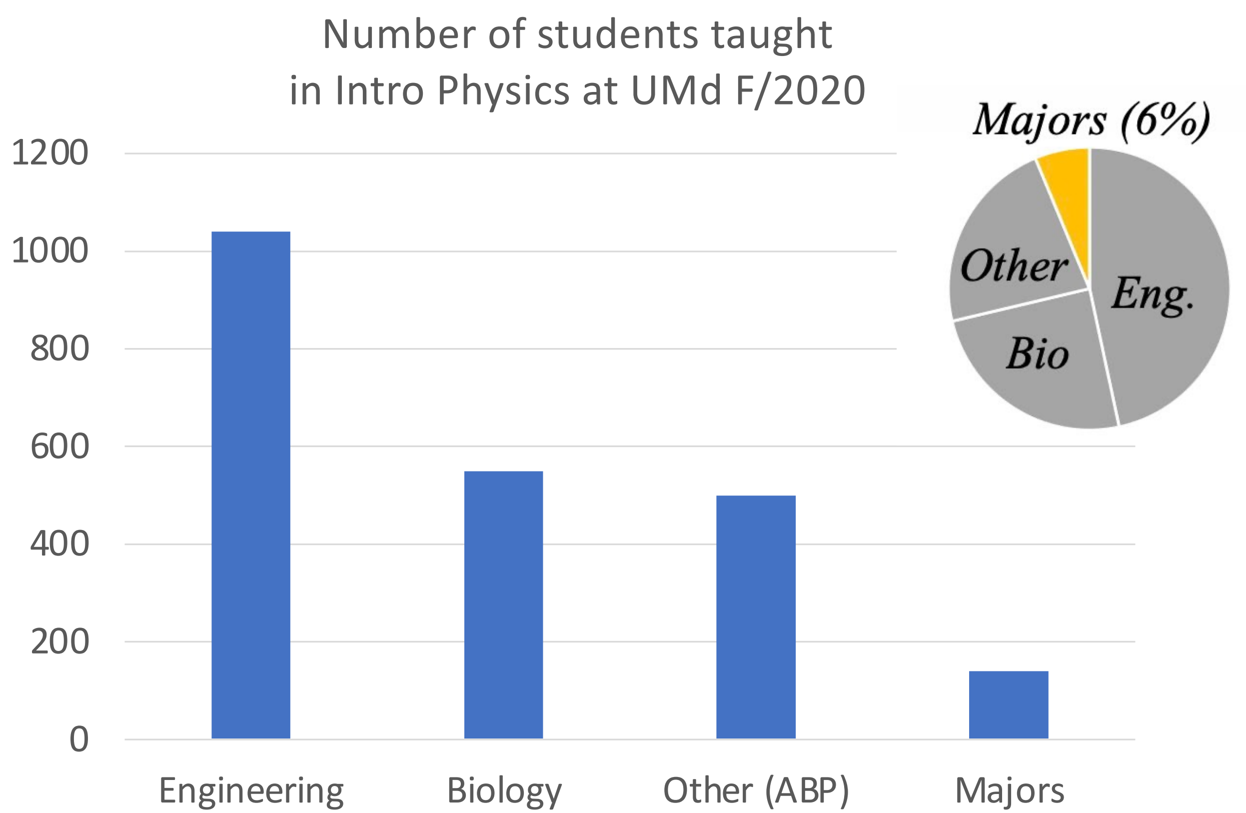
Work supported in part by NSF grants 11-22818, 15-04366, 16-24478, and a grant from HHMI.

A significant fraction of our students are **life science** majors.

- Often they are grouped as "other" and taught a highly generic "algebra-based physics".
- This rarely works well for them.
- Neither the students nor most life-sciences faculty perceive physics as having significant value.



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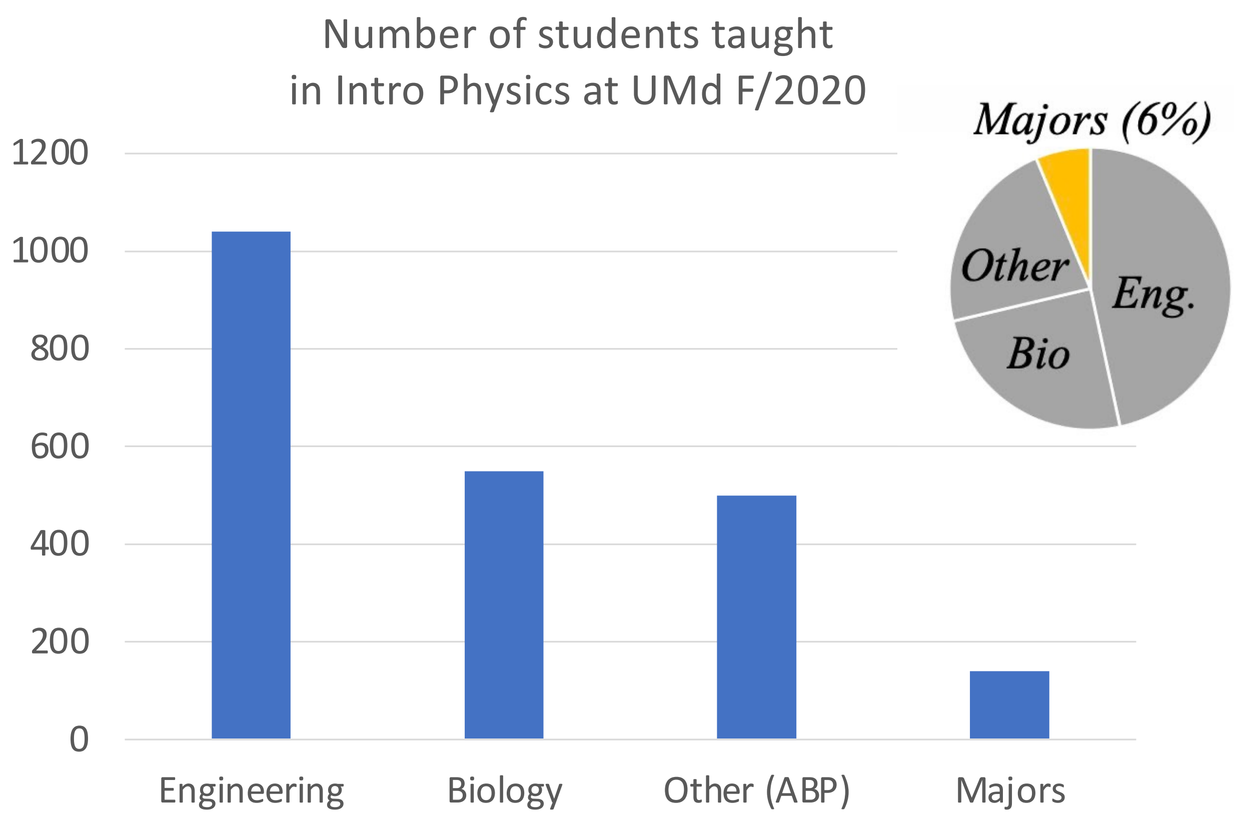
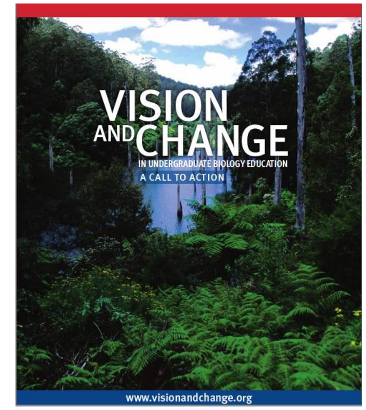
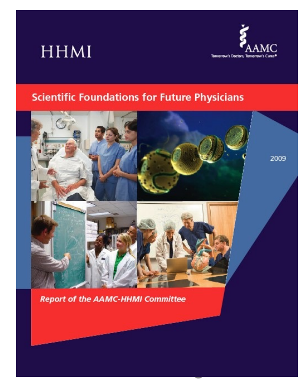


But…The Biology/Medicine Reports



- Leading biologists and medical schools have been calling for major reform of undergraduate instruction
- In 2009, AAMC (with HHMI) published *Scientific Foundations for Future Physicians* – a call for rethinking pre-med education in the US to
  - bring in more coordinated science – biology, math, chemistry, **and physics**
  - focus on scientific skills and competencies.



NEXUS/Physics

- In 2010, my research group won a challenge grant from HHMI and a research grant from the NSF.
- Our charge was to reform physics for life science students as part of a national effort to increase their skill development and multi-disciplinary strengths.

(National Experiment in Undergraduate Science Education – NEXUS).

E. F. Redish, et al. (17!) "NEXUS Physics," *Am. J. Phys.* **82**:5 (2014) 368-377. doi: 10.1119/1.4870386

Our context

- We created a special course for life science physics. We now teach about 600 life science students per term in a two-semester course.
- Both semesters are taught each term (including summer).
- We have moderately large lectures (~100-200)
- Classes include
  - Lecture 3 hrs/wk: often with clickers (N ~ 100-200)
  - 3 hours of recitation/lab per week (N = 24 per section)
  - Weekly homework, mostly online (using ExpertTA).
  - Individual hour exams but a common final.

An interesting opportunity for deep research!

- We brought together a large team of physicists, biologists, chemists, curriculum developers, and education researchers.
- We spent a year discussing what physics could do as part of a curriculum for biologists and health care students.
- We spent two years teaching the class in small classes, videotaping everything in sight and learning as much as we could from listening to (and interviewing) the students.
- This was a research project.
  We documented it in ~30 peer-reviewed papers.*

https://www.compadre.org/nexusph/course/NEXUS_Physics_Publications

What we found:
Barriers to interdisciplinarity

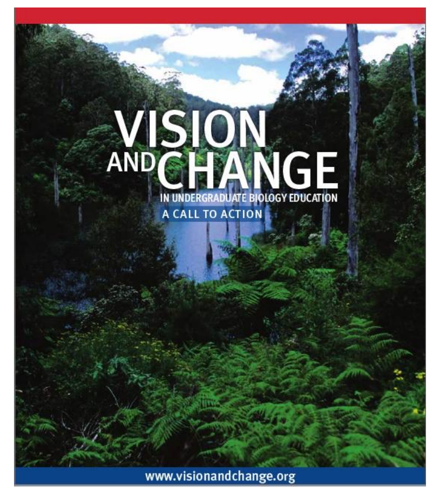
- Our initial negotiations immediately ran into a brick wall. Biologists and physicists had very different views of what to do.
- Many professional biologists saw most of traditional introductory physics class as useless and irrelevant – and our standard “we can apply physics to biological examples” as trivial and uninteresting.
- This resistance was found in students as well. Many expected physics to be useless. Most avoided taking it until they were seniors (when the grade wouldn’t count for med or grad school applications).

Adding value:
Learning to think with math

- The *Scientific Foundations for Future Physicians* study (SFFP) lists a number of competencies for students entering medical school that their undergraduate experience should prepare them for. This is the first.

> *Competency E1*
> *Apply quantitative reasoning and appropriate mathematics*
> *to describe or explain phenomena in the natural world. (SFFP p. 22)*

- The *Vision & Change* study from the biology research community has a similar goal for biology majors.



Learning to use math in science is important for life science students (and other living beings)

Physics is the best place to learn to reason about the world mathematically! 🤷‍♂️ However…

- Many non-physics STEM majors succeed in their intro level courses by memorizing facts, algorithms, and heuristics.
- Most have never seen an example where using math improves their professional understanding.
- We need to negotiate a change in student expectations

It's not just IPLS students! Chemists and engineers too!

Let’s add to our learning goals

- Students will learn to use math effectively in science, including being able to:
  - **Calculate** the value of simple expressions and solve simple equations when the equations are given.
  - **Work with equations in symbolic form** including manipulating them and interpreting them for physical implications.
  - **Generate appropriate symbolic equations** describing physical situations for which no explicit numbers are given.
  - **Reason about physical situations using symbolic mathematics** both qualitiatively and quantitatively .
  - ….

How often do we ask our IPLS students to do more than the first?

IPLS students often struggle with the math

- IPLS students are well know for having problems using math effectively in their physics classes.
- There is often active resistance to using symbolic math.
- This is sometimes assumed to be due to a lack of math skills (and sometimes it is), but often, something subtler (and more interesting) is going on.

*It's not just IPLS students! You see the same thing with engineers.**

These are the reasons we sometimes give up and settle for only trying to achieve the first learning goal.

* Torigoe & Gladding, Connecting symbolic difficulties with failure in physics, Am. J. Phys. 79:1 (2011) 133-140

Why is using math in physics hard for so many life science students?

The Math: Some observations

- My students often see math as inappropriate in biology.
  - As a result, they often resist thinking about the math.
- My students often find that despite A’s in calculus, they can’t do the math I ask them to do.
  - Even though the math is trivial algebra? What’s up wit
- My students are desparate to put numbers i and hate doing symbolic problems without numbers.
  - And they wan’t to drop the units, only putting them in at the end.
- My students prefer lots of plug-and-chug “practice problems” rather than a few “thinking problems.”

Any other instructors out there seeing this as well?

Meta-misconceptions

- While some students may indeed lack mathematical skills, many (in my experience) can “do the math” when it’s expressed *as* math.* (At least after a brief review.)
- More serious problems (that are not easy to remedy by a review class or by math ”drill-and-kill” activities) are students’ **inappropriate expectations** about
  - the nature of the knowledge they are learning (***epistemological misconceptions***)
  - the nature of what mathematical symbols are being used to represent (***ontological misconceptions***)

* Meltzer, King, & Jones, many AAPT talks (2016-2020)

"Epistemological misconceptions"

- The problems on the previous slide result from students bringing in **incorrect expectations about the nature of the knowledge** they are learning and about the role of mathematics in science.

- So. What do we do?
  - Give up and say, "these students just can't do it"?
  - Figure out what their problems are and see if we can teach them how to do it?

- If the second, we need to better understand the issues. Here's the key:

  The way we do math in science is very different from the way math is done in math. We **blend** physical concepts with math and this *changes the way we interpret the symbols*.

Meredith & Redish, Reinventing Physics for Life Science Majors, *Physics Today 66:7* (2013) 38-43

Really? Consider two examples.

A. 1 inch = 2.54 cm is a legitimate equation

3 second = 3 cm is not.

The numerical quantities we deal with in physics are typically NOT numbers, but stand for something physical that can be quantified in a variety of ways depending on choices we make.

B. The Electric field is defined by $E = F/q$ where q is the charge on the test (probe) charge. If q is replaced by $q/3$. E does not change.

The symbols in the equation are not arbitrary mathematical placeholders. F is not just a symbol; **it's the electric force felt by the charge q**. Changing q implies that F will change as well.

Redish & Kuo, Language of physics, language of math, *Science & Education* (2015-03-14)

Math in science is not the same as math in math

- Math in math classes tends to be about numbers (and the structure of abstract relationships). Math in science is not.
- Math in math classes tends to use a small number of symbols in constrained ways. Math in physics uses *lots* of symbols in different ways.
- The symbols in science classes often carry meaning that changes the way we interpret the quantity.
- In introductory math, equations are almost always about solving and calculating. In physics it's often about ***explaining*** **and** ***making meaning***.

When we do a derivation of an equation we are ***giving an explanation.*** *Intro physics students rarely understand that this is what we are doing.*

A particular problem with not blending

- It's not enough to recognize a concept belongs to a particular symbol. The blend has to be more contextual than that.
- We often use the same letter to represent multiple concepts. We figure out which is which by context and understanding the blend as describing a particular physical situation.

p = pressure
momentum
dipole moment

Q = charge
heat flow
volume flow

V = velocity
volume
voltage

T = tension
temperature
period

The blend

- The critical difference in the way we use math in science is that our symbols do not just represent numbers.
- In physics, symbols represent a **mapping** of physical meaning into mathematical symbol that **blends** together
  - our conceptual knowledge of the physical content and
  - our structural knowledge of mathematical relationships and processes.
- Making this blend is a non-trivial mental process that is rarely taught explicitly.

The blend has powerful implications

Having to do with the nature of scientific knowledge

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- Physical meaning can be represented by
  and organized with symbolic equations.
- We can reason about physical systems using
  abstract mathematical manipulations.
- If the starting point is physically correct and the math
  is done correctly, the conclusion should be physically correct.

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  - The physical world can be most conveniently described using quantities that have no direct sensory correlate but are best defined through mathematical structures.

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Can the blend be taught?

- Most professional physicists have never explicitly been taught about the blend. We learn it over many years — often (maybe especially for theorists) not until after they have completed their undergraduate training.
- Is it possible to teach this to our "I-only-have-to-take-one-year-of-physics" students in other disciplines?
- To do so, we probably need to have a detailed understanding of what "living in this blend" involves.

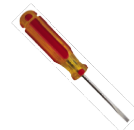
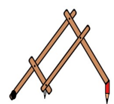
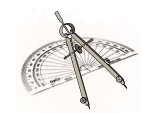
Teaching the blend: Epistemological issues

Teaching the blend

- One way to begin to approach teaching the blend is to be more explicit about identifying and teaching the core types of reasoning we use in the blend.
- I have created a **math-in-science toolbelt** with specific analytic and problem solving strategies.

- I teach these tools ("epistemic games") explicitly .
- Every time I use them (in class or in the text) they have an icon that appears to remind them that we are using that tool.

We can teach the blend through general purpose tools / strategies

Dimensional analysis		Reading the physics in a graph	
Estimation		Telling the story	
Functional dependence		Diagrams	
Anchor equations		Repackaging	
Toy models		Building equations from the physics	

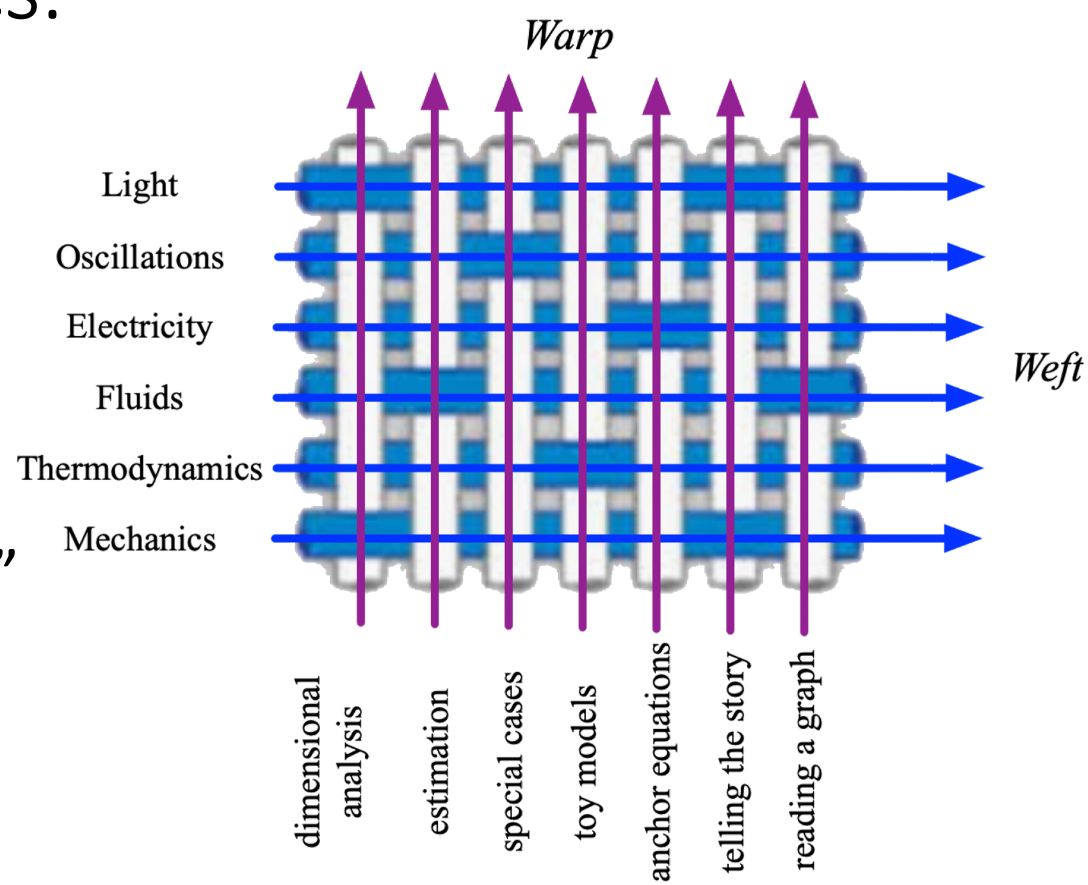
https://www.compadre.org/nexusph/course/Building_your_mathematical_toolbelt

Epistemic games (e-games)

- Strategies, not algorithms!
- Students have to learn the rules of the game (e.g., in "Estimation" you have to be able to justify why you chose the numbers that you made up and you can't keep more than 2 sig figs anywhere in the calculation)
- They have to practice in multiple contexts to learn how to use and adapt the tool to different situations.

Skills apply to all topics.

- Adding a focus
  on developing math skills
  doesn't add new topics.
- It modifies the way we teach
  topics we already include.
- It does require us to be "meta"
  — to be explicit about tools
  since students don't easily
  generalize.

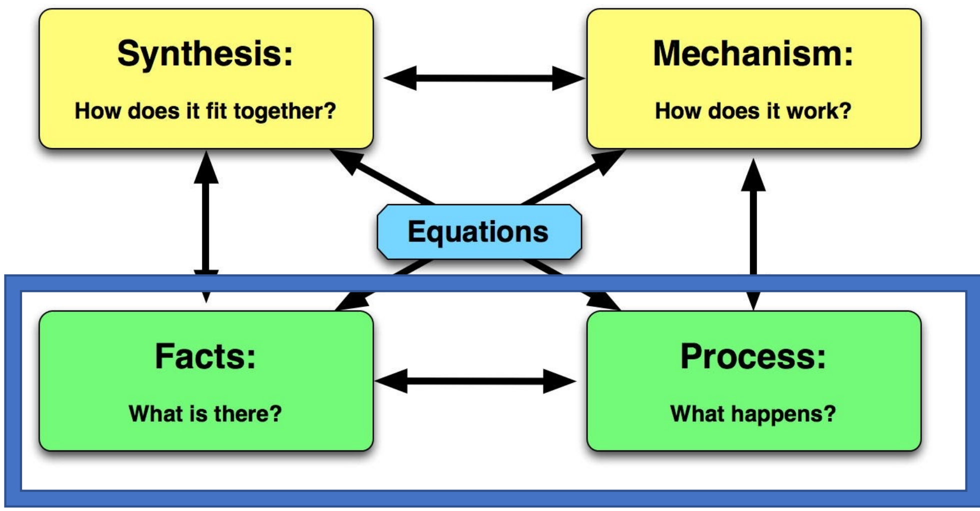


Redish, Using math in physics: Overview, to be published in The Physics Teacher (2021)

Negotiating a new epistemological stance

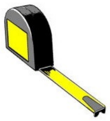
Epistemologies: The nature of scientific knowledge

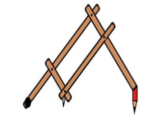
- Knowledge in science is not just a collection of ***facts*** and ***processes***.
- Much of the value of scientific knowledge is in understanding ***mechanism*** (chained causal reasoning)
- — and in ***synthesis*** (how powerful principles integrate large blocks of knowledge).



- Symbolic mathematical equations help to organize and connect these different knowledge structures.

Flash-card science!



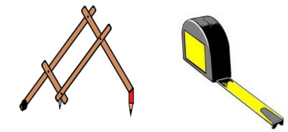


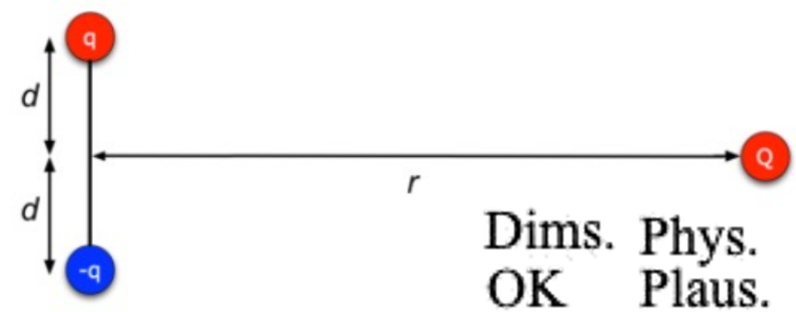
1. Dimensional analysis / functional dependence

- Most quantities we use in physics are quantified as a result of *measurement*. They therefore have **dimensions** (mass, length, time, charge, temperature) and **units**.
- Learning to see the dimensionality of symbols is a first step in building the blend of physical concept with math.
- Learning to pay attention to the **functional dependence** of a symbol (how it enters an equation) is a powerful tool to build student perceptions of **authenticity*** of symbolic math .

* Watkins & Elby, Context dependence of students'views about the role of equations in understanding biology, CBE-LSE 12 (June 3,2013) 274-286.

An electric dipole consists of a pair of equal and opposite charges separated by a small distance. Consider a dipole consisting of charges $+q$ and $-q$ separated by a distance $2d$. The dipole moment is defined as $p = 2qd$.





Which of the formulas at the right is the correct law for the force between p and Q for large r?

Evaluate if the dimensions of each expression are correct and whether the functional dependence is physically plausible.

(For example, since we know the force gets weaker as Q gets farther away, the distance r can't be in the numerator.)

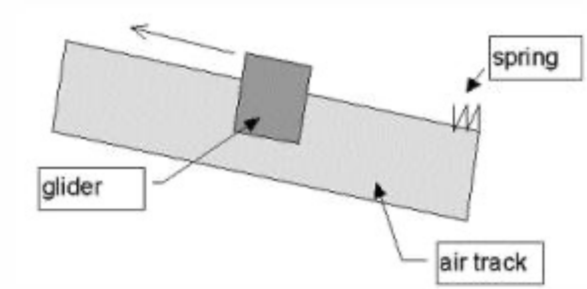
	Dims. OK	Phys. Plaus.
A. $F = k_C\left(\frac{p^2}{Qr^4}\right)$		
B. $F = k_C\left(\frac{Qp}{r^2}\right)$		
C. $F = k_C\left(\frac{Qp}{r^3}\right)$		
D. $F = k_C\left(\frac{Q^3}{pr}\right)$		

2. Reading the physics in a graph

- Students often frame the creation of a graph as an *answer* (something the teacher has asked them to do) rather than as a *tool* (a way of understanding something physical).
- The fact that each graph codes many different bits of information and we use multiple graphs to describe the same physical situation makes it a GREAT way of helping students learn to build the blend.

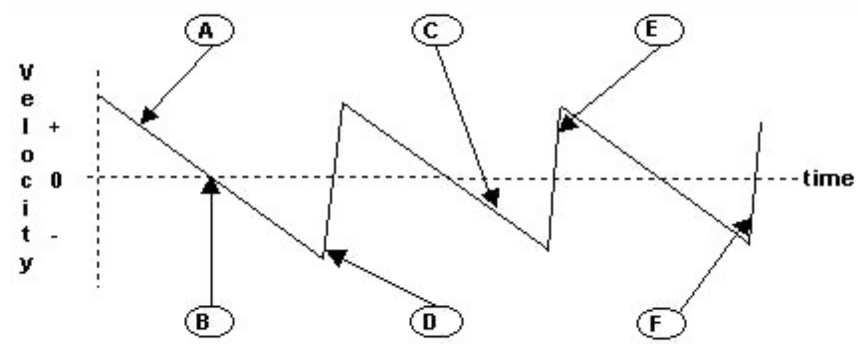
A tilted airtrack has a spring at one end. A cart on it is pressed against the spring and released. The cart bounces up and down.

The graph represents the velocity of the cart as a function of time starting at the moment of release. Positive is to the left of the diagram.



Which of the letters on the graph can identify an instant of time
at which the cart is

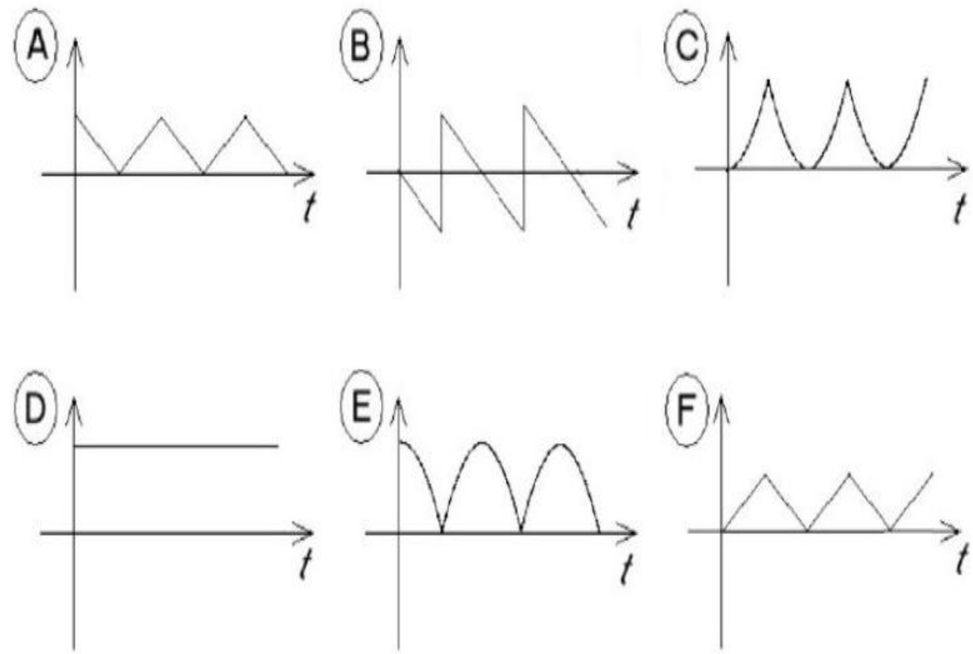
1. instantaneously not moving
2. in contact with the spring
3. moving down the track
4. at its highest point on the track
5. has an acceleration of zero.



A superball is dropped from a height of 1 m and bounces a number of times before it is caught. Below are shown graphs of some of the physical variables of the problem.

Which graph could represent

1. The velocity of the ball
2. The kinetic energy of the ball
3. The gravitational potential energy of the ball
4. The momentum of the ball
5. The total mechanical energy of the ball.
6. The position (height) of the ball

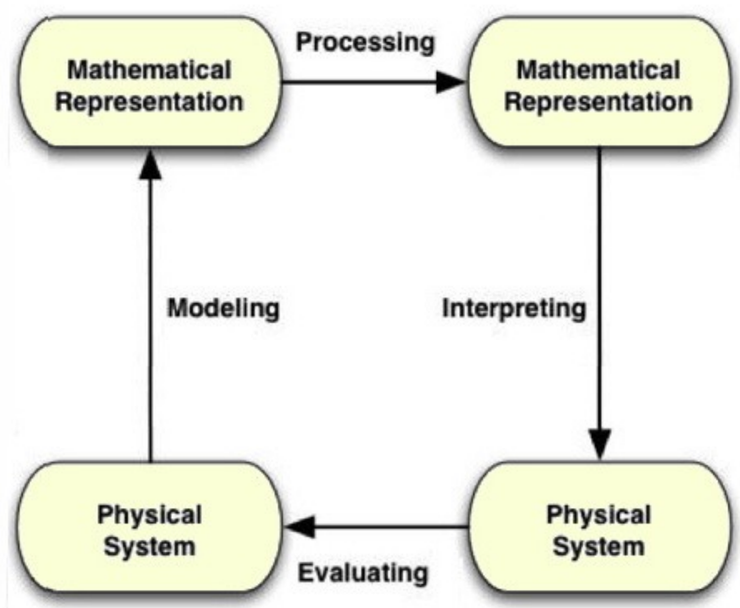


3. Toy models

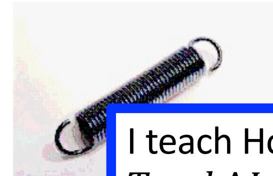
- One of the most powerful tools we use in physics in learning to make sense of the world with math are ***toy models***.
- We consider the simplest possible example that shows a phenomenon and beat it into submission — until we understand it completely.
- These are extremely valuable starting points for building more complex models of realistic situations.
- The problem is that biologists (and engineers) tend to want to focus on realistic situations. This leads them to see toy models as bogus and irrelevant.

We need to be explicit about toy models, motivate them, and consider corrections.

- Using toy models is a great start for learning how to model more realistic situations.
- But even a toy model involves multiple steps and “the blend”.
- If we only give problems that involve the top leg, students think “it’s just math” and ignore the critical steps of deciding
  “What’s important?”
  “Is that good enough?” and
  “What does it mean?”



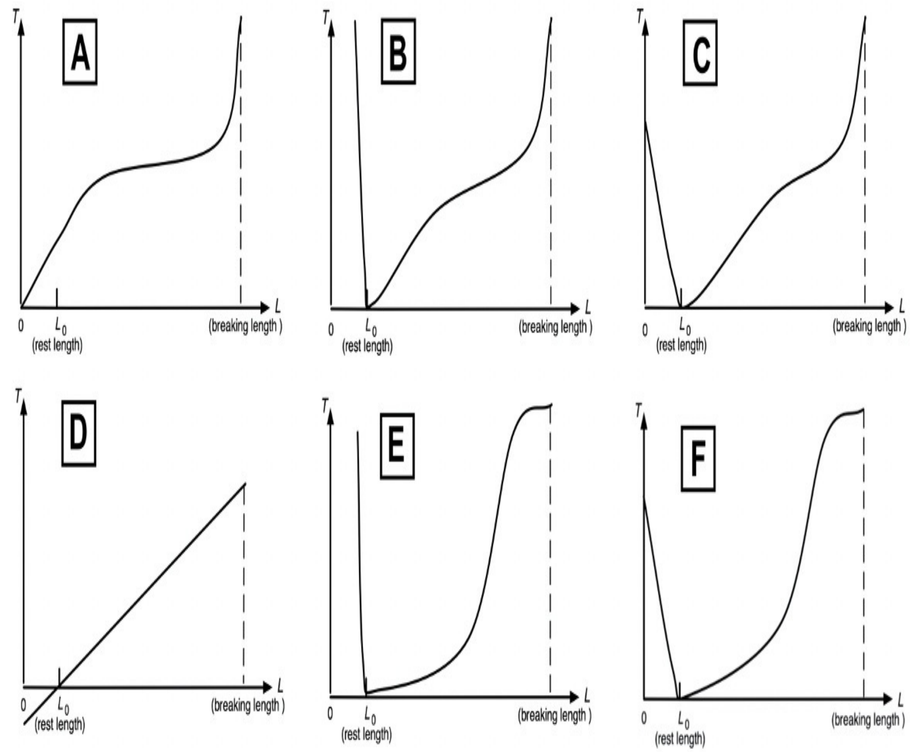
Real springs only follow the Hooke's law model for small displacements around their rest length. Which graph might represent T vs L for a real spring?



I teach Hooke's law as $T = k\Delta L$ rather than as "$F = -kx$" to focus on the physical blend.

A real spring behaves as follows:

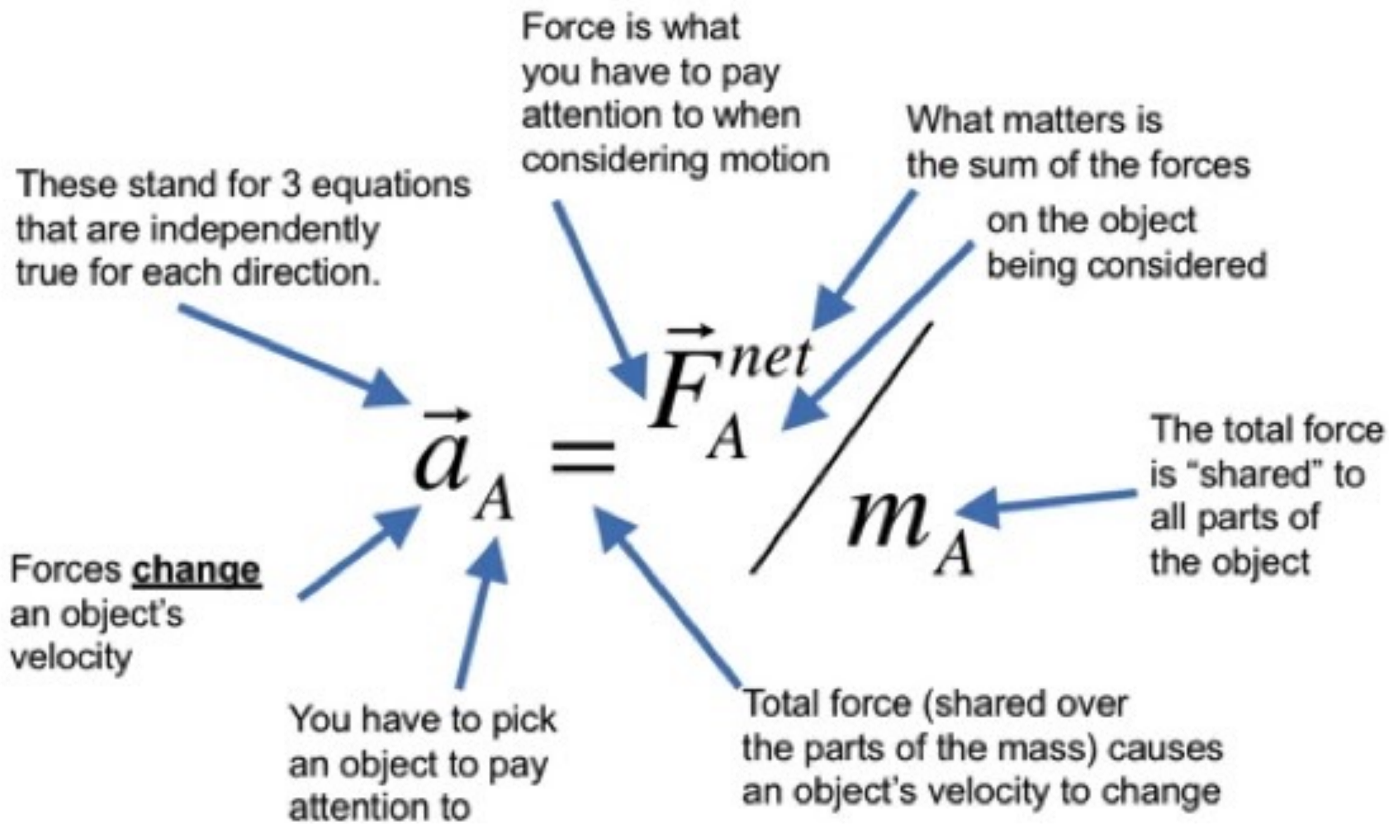
- stretching from its rest length, it obeys Hooke's law for a small stretch.
- As you stretch it further, it gets stiffer as the coils begin to bend.
- Eventually it straightens out into a long straight wire which is very hard to stretch.
- If you keep pulling harder, the wire suddenly stretches easily and breaks.
- If you try to compress it, the coils get pushed together and you can squeeze very hard without getting much compression.



4. Anchor equations

- Anchor equations provide stable starting points for thinking about whole blocks of physics content.
- They are the **central principles** that provide a foothold — a starting point for organizing our understanding of an entire topic.
  - Coding for conceptual knowledge
  - A starting point for unpacking other relevant knowledge
  - A starting point for solving problems

An example: Newton's 2nd law organizes conceptual knowledge about how force affects motion



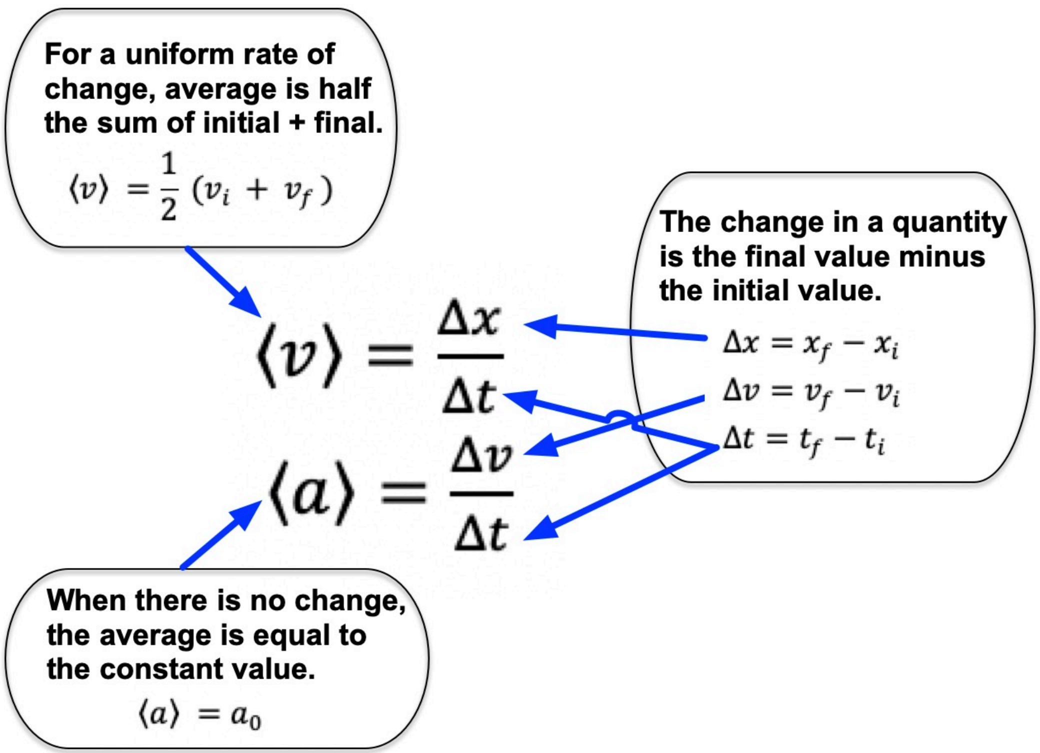
An example: Kinematics concepts (blended)

- The average velocity is given by the change in position (How far did you move?) divided by the time interval (How long did it take to do it?).
- The average acceleration is given by the change in velocity (How much did it change?) divided by the time interval (How long did it take to do it?).

$$\langle v \rangle = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt}$$

$$\langle a \rangle = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt}$$

Unpacking the anchor equations



Using these equations, problems are solved by:

1. Identifying relevant quantities in the physical problem (mapping physics to symbol)
2. Calling on relevant math concepts and matching them to the identified physical values
3. Identifying knowns and unknowns and manipulating to get solvable equations.
4. Solving the problem.

Starting with the anchor equation

- Helps students learn to
  match the physical situation with math symbology.
- Get used to the idea that
  they can do simple algebraic manipulations.
- Learn that
  they can set up equations from a physical situation

  They don't only have to use equations
  that someone else gave them
  (or that they searched on the web).

Teaching the role of math in physical ontology

Ontologies

- New ontologies may be defined mathematically. Students may not be used to quantities where the fundamental structure of "what it is" is mathematical
  - Example 1: The conceptual shift from Electric Force to Electric Fields
- Complex concepts may require dynamic or blended ontologies (e.g. wave/particle duality)
  - Example 2: Metaphors for energy and chemical bonds*

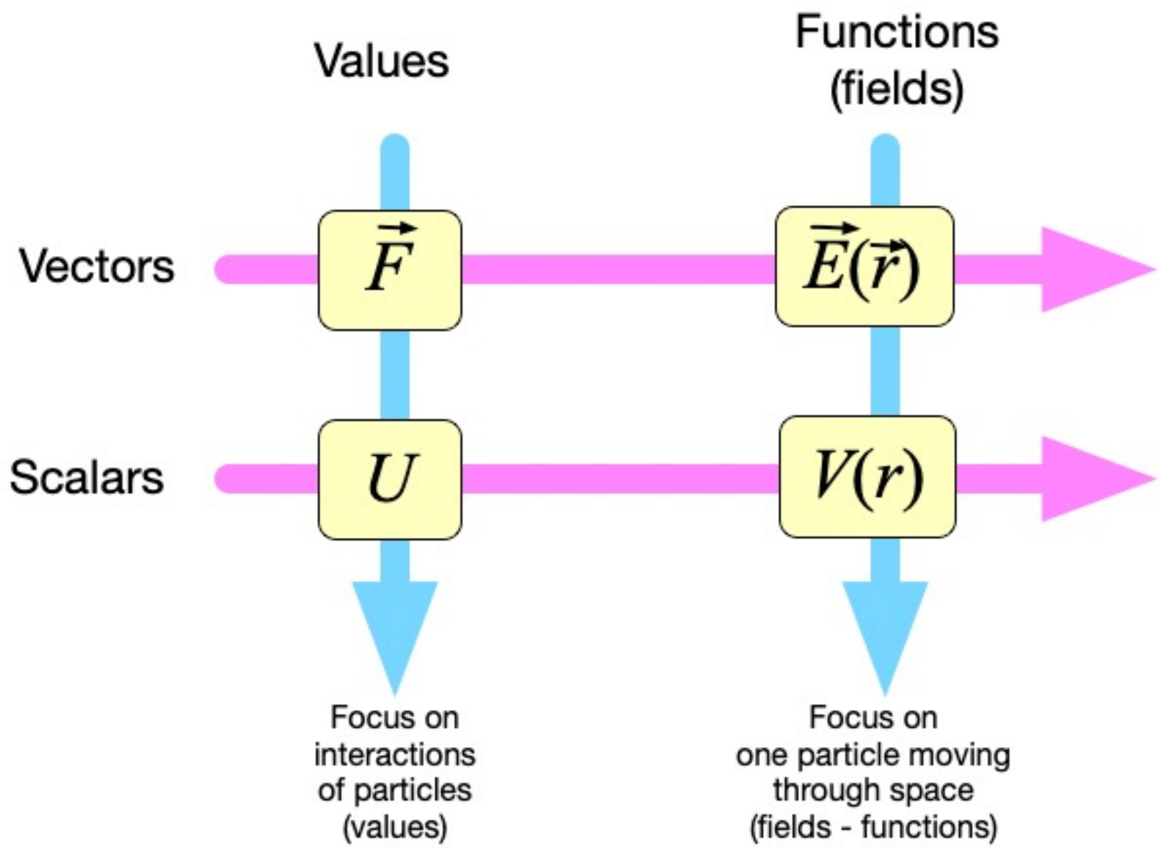
* B. W. Dreyfus, et al., *Phys. Rev, ST-PER* **10** (2014) 020108

Ontological example 1:
Electric concepts

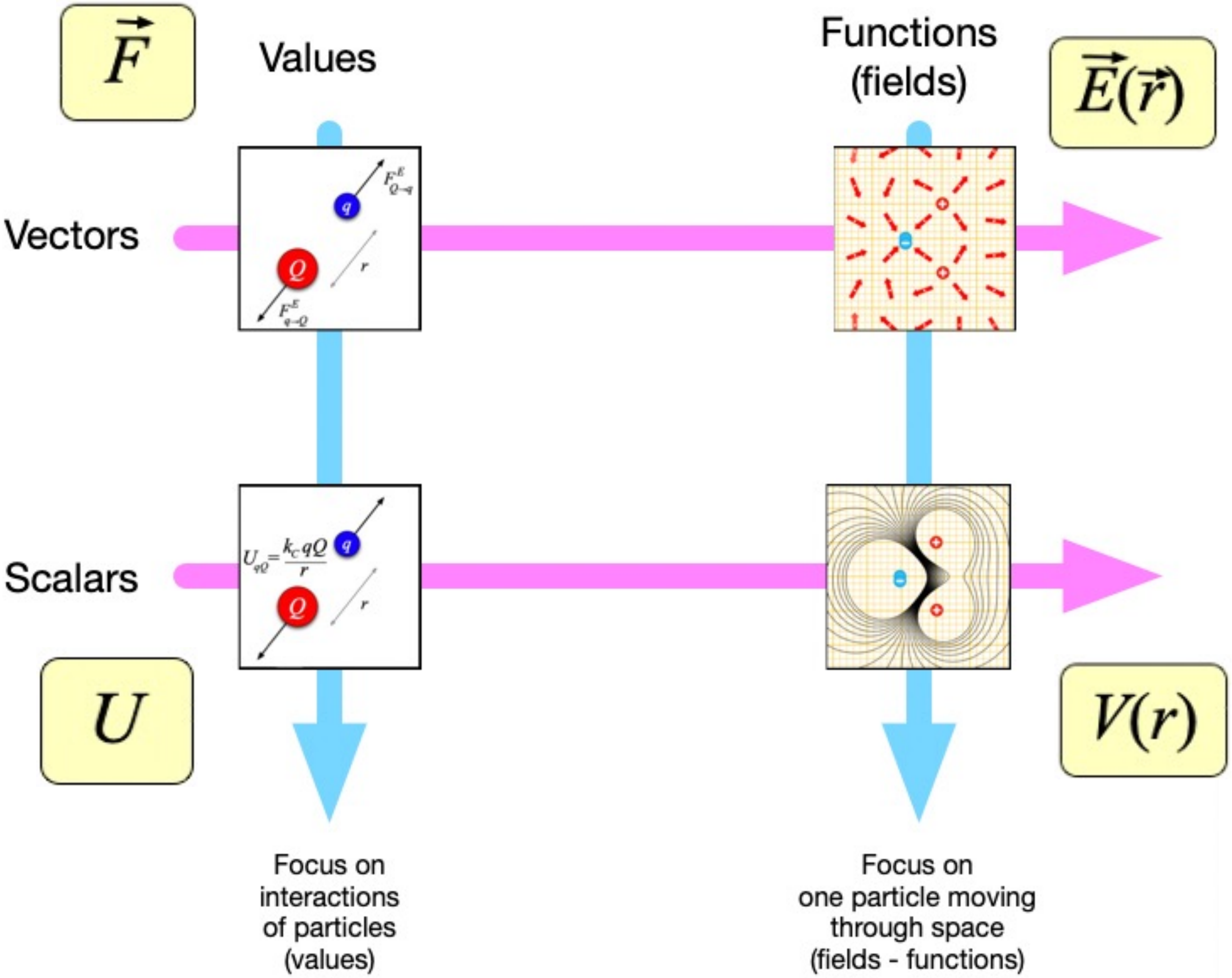
Example:
Electric quantities defined mathematically

- Often in physics we define "what something is"
  (its ontological classification) by what it is mathematically.
    - How it transforms under changes
        - Is it a vector or scalar?
        - What are its dimensions (units)?
    - What is its mathematical character?
        - Is it a value or a function?
        - Is it a parameter or a variable?
- Many of our constructs are like this.
  It can be hard to build a physical mental construct
  if you're not used to doing that sort of thing.

There are four crucial electric concepts of differing mathematical character



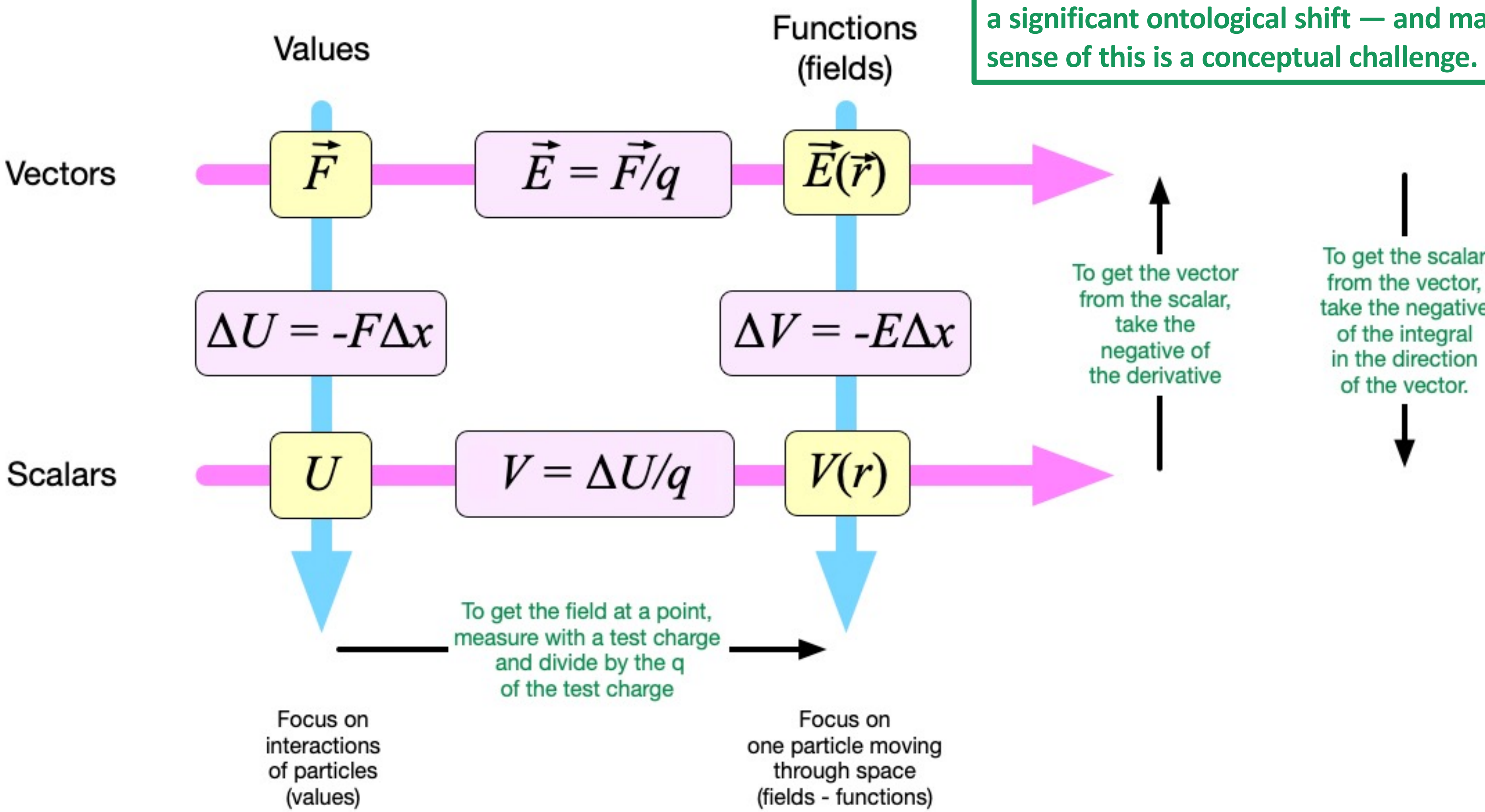
* Based on “The Big Square”, a lesson by Vashti Sawtelle and Ben Geller, adapted from The Modeling Collaboration, 2012



Diagrams that we use to construct each, illustrate the different characters of the quantities.

Connection via equations

These equations are not just showing ways to calculate a new quantity. Each represents a significant ontological shift — and making sense of this is a conceptual challenge.



Concluding remarks

Whatever class you are teaching,
whatever constraints you are operating under

- There are lots of opportunities to be creative
  and to change things in ways that will improve
  student learning and their appreciation of our classes.
- I was surprised and delighted by how many deep insights
  I gained into physics that I thought I knew
  — and how much new and interesting physics I learned
  in order to reach my students
  in a place that was meaningful to them.
- Working with biophysics colleagues was a great help.
  I expect you can find engineering physics colleagues as well.

Using math in science: Take away messages

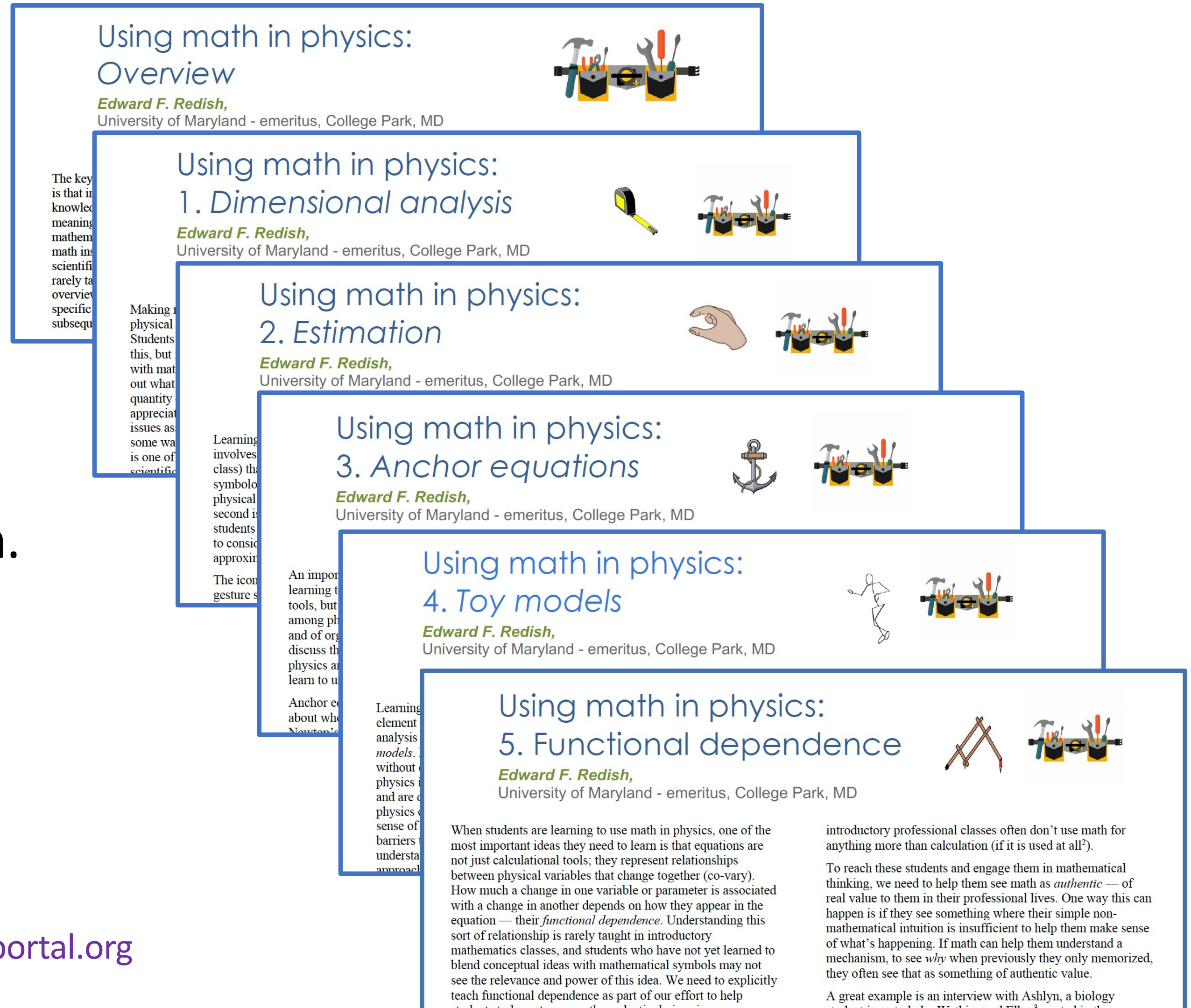
- Using math in science is not just using math.
  There are a lot of new skills
  (blending physical meaning with math)
  that students need to learn.
- Some student difficulties are epistemological —
  inappropriate expectations about
  the nature of knowledge in physics.
- Some student difficulties are ontological —
  difficulties with seeing mathematical structure
  as carrying physical meaning.

This is the tip of the iceberg!

- There remains much room for research, both empirical and theoretical.
- There is much room for new kinds of curriculum focused on these issues.

Resources

- I am preparing a series of short modules on "Using Math in Science" with overview papers on each e-game and collections of sample problems for teaching them.
- So far I've completed 6.
- These are available on arXiv and at



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