Do External Mathematical Tools Affect How Students Think About Physics?

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Outline

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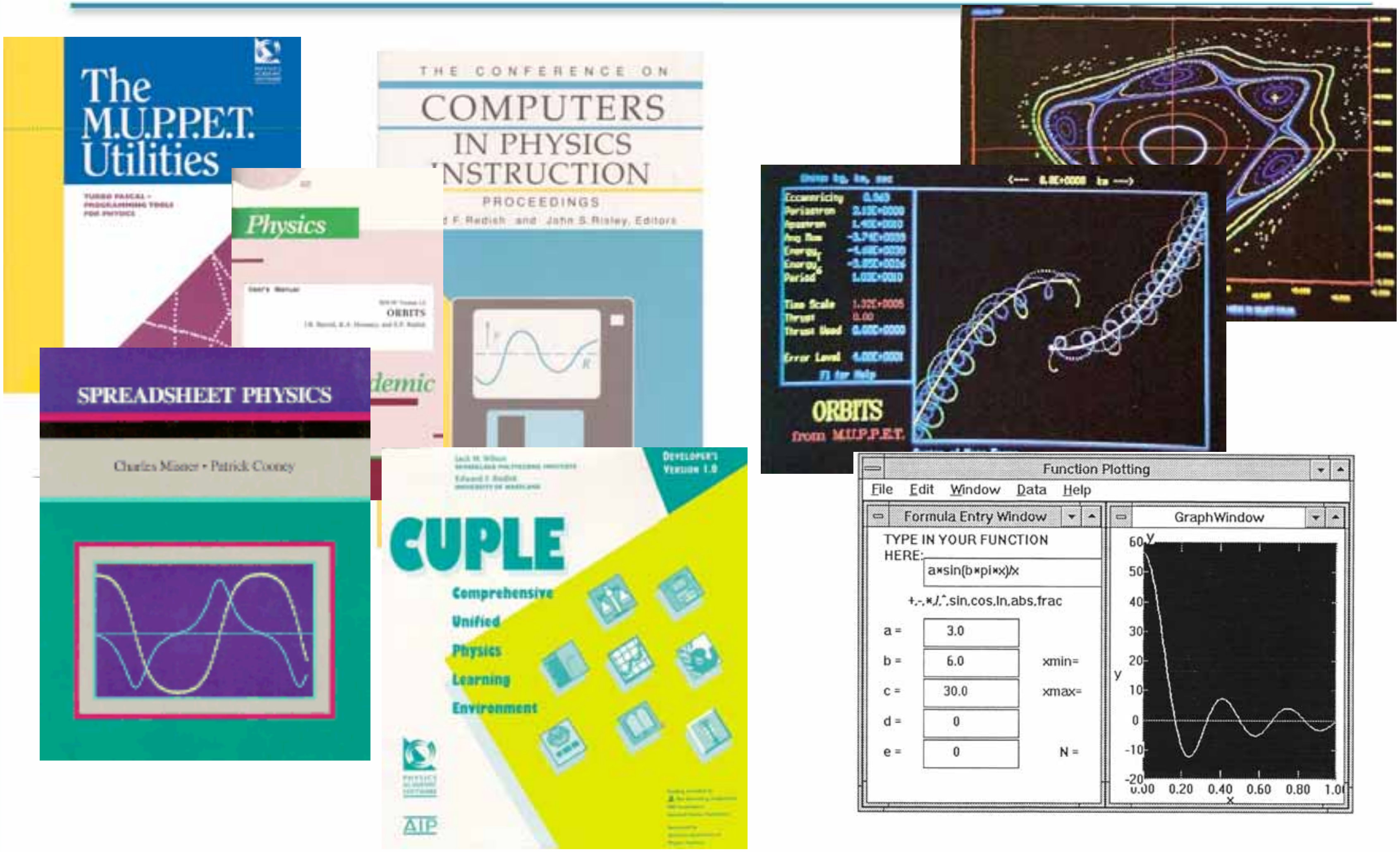
- Cognitive modeling
- Framing the study
- Epistemic stances:
  A case study from Mathematical Physics
- Using Mathematica:
  A case study from Quantum Mechanics
- Conclusion

A Personal Historical Perspective

Some History

- I got into working on physics education in the early 1980's with the appearance of the personal computer.
- My goal was to use computing to allow physics majors to
  - get into research sooner;
  - See contemporary physics sooner;
  - Use computing as a "hands on" way of learning to think about the physics.

Lots of computer stuff!



I learned a few things while doing this work

- Some positives
  - Many students were motivated by doing computer projects as freshman
  - The number of students doing research later increased dramatically.
- Some negatives
  - They didn't seem to improve their understanding of the physics.
  - I wasn't sure that I understood what *I* wanted them to get out of this work – and how to evaluate whether they got it.

A modern perspective

- Tools have changed!
  - We used Basic and Pascal in DOS!
  - We now have OOPS and Java and powerful pre-programmed tools and simulations.
  - Modern calculators can graph, take data, solve equations, and do integrals analytically.
  - We have powerful symbolic manipulators.
- An we have learned a lot about the complexity of learning a complex subject such as physics.

Thinking about thinking in the context of computation

- Can we use what we have learned about student learning to better understand our goals, our students' difficulties, and our instruction in advanced physics?

Cognitive Modeling

What I've learned from teaching: Condensing 40 years into 30 seconds

- Students are (too often) satisfied with a one-step argument.
- Students find it hard to imagine that something they remember might be wrong.
- Students can know stuff that they need that they don't know they need.
- Students can have a lot of trouble seeing the physics for the math.

Thinking about thinking: Some (fairly obvious) truisms

- Thought is a dynamic and fluid process.
- You can espouse different (and even contradictory) ideas at different times.
- You aren't aware of everything you know at any given instant.
- Sometimes it's difficult to recall things you are certain you know.
- Sometimes you don't realize that something you know is relevant to an issue at hand.

Why cognitive modeling?

- We would like to develop a framework so that we can talk about these issues.
  - We want to establish benchmark knowledge that we can trust.
  - We want to be able to build on what we know and build a stronger knowledge base about teaching and learning.
  - We want to be able to be able to be able to teach new teachers what the critical issues are so that they don't have to take 40 years to get it!
  - We want to be able to design our curriculum reforms so that they are effective.

Components of a theoretical framework for cognitive issues

- Working Memory (WM)
  - What's active in your brain at a given time but
    - ***It can't hold much***
    - ***It has limited processing power***
- Long Term Memory (LTM)
  - Can store lots of data for long times but
    - ***You can't find anything in it***: LTM is not indexed (you can't Google your brain!)
    - ***You can't trust what you find***: LTM is not a perfect copy of experience but is rebuilt using standard elements.
    - ***Access to memories are through associational chains***

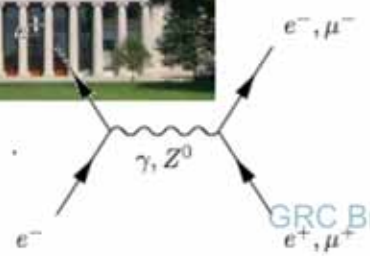
Basic Principles: The Resource Framework

- ***Activation*** – When your thinking about something, bits of your brain are "on".
- ***Association*** – Access to info in LTM is through "spreading activation" that depends upon links and associations.
- ***Compilation*** – Repeated activation of different elements together can merge them into what appears to be a single undifferentiated "bit" of knowledge.
- ***Control*** – Not everything is used at once. Knowledge about situations is used to open (or suppress) access to relevant knowledge.

An association pattern

Who is Andy diSessa's thesis advisor?

Roman
Jackiw!



Implications

- It's important that students learn stuff, but how and when they can use what they know is as important as what they know.
- What seems "obvious" to us, may be complex and not obvious to students.
- Recognizing when to use different kinds of knowledge can be hard to learn.
- A lot of important "stuff" may be happening when we learn physics that we are not aware of and that we don't explicitly teach.

The Study

Studying learning math in physics in upper division classes

- Classes: Physics Majors
  - Methods of Mathematical Physics
  - Quantum I & II
  - E&M
  - Classical Mechanics
- Data: Ethnographic Video
  - Homework groups (~100 hours)
  - Interviews (~25 hours)
  - Classroom (~25 hours)

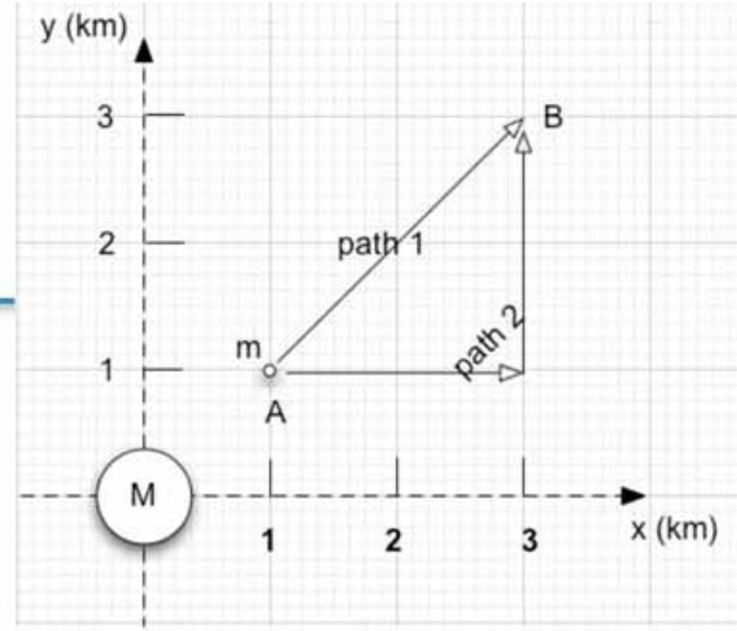
Focus on access and control

- How do intermediate level students use their knowledge?
- Framing:
  - When we enter a situation, we quickly assess it and activate our expectations of what behaviors are appropriate.

Social
Structural
Epistemic

An Example

- A rocket is taken from a point A to a point B near a mass m. Consider two (unrealistic) paths 1 and 2 as shown. Calculate the work done by the mass on the rocket on each path. Use the fundamental definition of the work



$$W_{A\to B} = \int_A^B \vec{F}\cdot d\vec{r}$$

not potential energy. Mathematica may or may not be helpful. Feel free to use it if you choose (though it is not necessary for the calculations required).

Some discussion

Transcript 1

What's happening?

- During this discussion the two students are talking at cross purposes.
- They are each looking for different kinds of "proofs" than the other is offering.
- They use different kinds of "warrants" (reasons) to support their arguments.
- Eventually, they find mutual agreement – after about 15 minutes of discussion!

Epistemic stances

- In observing many hours of mid-level students working together, we see them using four kinds of epistemic warrants.
  - Calculation
  - Physical Mapping
  - Invoking Authority
  - Math Consistency
- The more sophisticated students can blend these approaches, but even good students often “get stuck” in a stance.

An Example

- Imagine two non-interacting particles, each of mass *m*, in the infinite square well. If one is in the state ψ_n and the other is in state ψ_m orthogonal to ψ_n, calculate $\left\langle (x_1 - x_2)^2 \right\rangle$, assuming that
(a) they are distinguishable particles,
(b) they are identical bosons, and
(c) they are identical fermions.

D. Griffiths, *Intro. to Q.M.*, prob. 5.5

Student response

- We observed a group of 6 students working on this problem. At some point, someone realized they had to evaluate integrals of the form

$$\int x_1^2 \left|\psi_n(x_1)\right|^2 dx_1$$

or more explicitly

$$\frac{2}{L}\int x^2 \sin^2\left(\frac{n\pi x}{L}\right) dx$$

They turned to Mathematica to do so.

Some discussion

Transcript 2

What's happening?

- What we see in the above transcript (and in more of it that has been omitted) is that the students
  - go to *Calculation* mode
  - remain in *Calculation* mode to seek repair when the hit a variety of roadblocks
  - do not choose to consider that they might be doing the wrong calculation by validating with *Physical Mapping*.

Is this typical?

- In observing dozens of hours of video of students working on upper-division homework problems we note:
  - Students often have trouble blending different epistemic stances effectively.
  - Getting stuck in Calculation is common.
  - A few more sophisticated students blend stances effectively and well.
  - The presence of powerful tools like Mathematica is often associated with extended Calculation “stickiness”.

What are the implications?

- We assume that students learn to blend the physics and the math a bit at a time, as we did. (Is that the way we did it?)
  - But this may be a filter rather than a normal occurrence.
- We often don’t stress the mixing of epistemic modes in our instruction.
  - Even if we do, students may not hear it.
  - Even if we do, we may not have good ways to evaluate it.

Implications for computation

- We might be able to teach good physical computation more effectively if we are more explicit in helping students see how to integrate physical knowledge and constraints with their computation.
- We need to not only develop the computational tools to facilitate students getting the results we want, but to develop the evaluation tools to show the students are learning to do physics with computation and not just computing.

Implications for simulation

- Simulation works the other way; it encourages physical mapping and discourages calculation.
  - Students often “calibrate” rather than figure out what a simulation is telling them.
- We need to develop both lessons and evaluations that help students learn what we want them to learn from simulations.

Conclusions

- We have thought about
  - Associational structures
    (what knowledge they link to what other knowledge in different contexts)
  - Control structures
    (how they perceive context and thereby determine access to their knowledge)

and this has brought our attention to the importance of elements of which we were aware but traditionally do not stress.

More info

- More information on this work is available on the website of the UMd Physics Education Research Group.
  - http://www.physics.umd.edu/perg

- Tom Bing's dissertation will be available at
    - http://www.physics.umd.edu/perg/dissertations

  at the end of July.