[00:03:11.29] TA tells students to put their homework on the table in front

[00:05:07.01] Facilitator introduces the Tutorial - today I recommend using the white paper to draw - use it as a way to have everyone contribute and to make the limits. Put your names in the corners so we can see them. If you have questions, just raise your hands, but otherwise we're going to walk around and bug you.
[00:06:03.10] Go ahead and get started

[00:06:13.13] I place the microphone and they smile

[00:06:14.26] That's pretty good I suppose. I tried

[00:06:22.26] It looks more like a manatee or something, that's fine

[00:06:32.11] the two tables makes it difficult to draw

[00:06:38.13] yeah, it's tough

[00:07:48.26] Felix: But it's final minus initial, so, you'd draw it from B to A

[00:08:36.21]

[00:10:00.24]

[00:10:51.02]

[00:11:19.11] Felix: Is the guy going at constant speed around the track?
TA: Uh, we don't know
S: You don't know?

[00:11:56.08]

[00:12:56.09]

[00:13:31.04]

[00:14:45.08]

[00:15:37.20] When we subtract vectors, when we do final minus the initial, the
[00:15:40.07] our product, where we do put that - our answer, where... from B to A? or A to B? Is it the same every time?

[00:15:47.21] If you're going to put the tails together, you go initial to final, if you put the tails together

[00:15:53.08] Because I always think that you're going to have A, plus the change to get B

[00:15:57.20] OK, yeah I got it

[00:16:00.06] TA: You just got three explanations for the same thing

[00:16:18.21]

[00:16:57.17]

***START
[00:17:15.20] Felix: Where are you drawing the angle?
[00:17:20.02] S3: I drew it, like between, like, right here, this theta here
[00:17:24.26] Felix: Yeah, but, is it the tail of vB?
[00:17:28.19] Oh the tail of, never mind, I just read it wrong.
[00:17:33.24] S3: Now, what's the angle?
[00:17:36.05] S4: Less than 90,
S1: Yeah
S4: because it looks like it
[00:17:44.09] S4: Yeah, it is less than 90. Because a 90 degree change would be when it's at this point,
S1?: Yeah, it would have to be perpendicular
S4: and not that point. Less
[00:17:50.27] Felix: As point B is chosen to lie closer and closer to point A, does the angle increase, decrease, or remain the same?
[00:17:55.24] S1: Increase, right?
[00:18:06.22] S4: As point B is chosen to lie closer to point A...
Felix: So right now
[00:18:12.14] S1: Wouldn't you just move this right here
[00:18:15.04] Felix: So right now it's like this, and
[00:18:22.02] S4: Wait, between what? We're looking at the angle...
[00:18:28.03] Felix: We've got a pencil here
[00:18:25.18] S4: Oh nice
[00:18:26.24] Felix: There (Uses two pencils to demonstrate) Getting closer... getting bigger
[00:18:33.29] Felix: Because
[00:18:33.27] S4: Oh true.
Felix: when it gets right next to it, it's 180 degrees
[00:18:36.22] S1: Yeah
S4: Yeah
***END

[00:19:40.01] S4: There's always acceleration..

[00:19:49.27] The smallest acceleration it will have is right here at the flattest part of the oval, so it will never be zero
[00:19:55.19] But just because something is reducing doesn't mean it's approaching zero

[00:20:20.27] S4: I think it does approach zero. Because if you choose two different intervals, the change is so small
[00:20:35.14] Facilitator: What happens to the magnitude our displacement vector? Oh, you skipped all the way through...

[00:20:55.05] F: So how about your average velocity vector, which way does it point?
[00:21:03.10] Towards the center. Well, center-ish, because we're doing the average, not the instantaneous

***START
[00:21:20.27] Facilitator: So it approaches 180, or it approaches what... so let's, let's do a drawing. Can I do a drawing?
[00:21:31.06] So let's say this is my velocity vector, vA, right?
[00:21:36.20] (Pencil tip breaks) Oh!
S1?: (Inaudible)
(TA laughs)
[00:21:40.16] and let's say that my... well let me do it in a slightly more exaggerated way. Here's vA,
[00:21:54.27] and here is vB'. OK?
[00:22:01.20] what's the angle here?

[00:22:06.22] S1: Oh, it's approaching...
S4: It's 90 degrees
[00:22:06.00] TA: (Looks at Felix and S3) Yeah? You see that?
[00:22:09.03] TA: So what happens as the... as this angle, this angle gets smaller and smaller as these things approach each other, in the limit that B' approaches A, these things
[00:22:16.29] start to align to each other, right? And in the infinitesimal limit, the time goes, limit of time, goes, delta t goes zero
[00:22:25.23] delta v also goes to zero but the direction of that
[00:22:30.22] S4: Is 90 degrees
TA: Yeah, it has to be that
[00:22:33.05] S1/S4?: Yeah. Thank you
TA: OK? Is that cool?
S1/S4?: Yeah... pretty cool.

[00:22:53.04] S4: OK, yeah, so the magnitude of the average acceleration approaches zero... yes
S: (erasing)
[00:22:58.01] S4: Because change in velocity approaches zero

[00:23:07.06] S4: The instant acceleration is directed towards the center... the center of the oval ... (YAWN)
[00:23:30.09] S3: So, the acceleration is always towards the middle of the circle? Right?
[00:23:33.21] Felix: Dude, it's not a circle, it's an oval
[00:23:35.12] S3: Or the oval, sorry, or the ellipse
[00:23:36.20] Felix: So wait, would it be of that, or would it be, like, whatever arc this is
[00:23:40.15] S3: Yeah, that's true
[00:23:44.15] S3: I think it would still be to the center but these would just be longer and these shorter
[00:23:46.17] S1: Yeah, I like that
[00:23:48.00] Felix: Oh, cool
[00:23:52.08] S3: So, what did you put, did you put, pretty much... yeah the same direction
[00:23:56.23] Felix: Yeah
[00:23:59.01] S4: (comes back from his yawn and puts his hands on the table) OK. Draw a diagram that shows the velocity and acceleration at point A... draw a diagram...
[00:24:16.20] S4: OK. Draw a diagram that shows the velocity at point A... vA... A, A
[00:24:32.23] S3: I'm not really sure how to get the acceleration to work, you just point it towards the center of the circle?
[00:24:36.01] S4: It always points towards the circle, the center of the circle
[00:24:40.02] S3: That's the instantaneous acceleration?
[00:24:43.07] S4: Or wait. Wait, wait, wait.
[00:24:47.18] Felix: Instantaneous acceleration?
[00:24:48.29] S4: Well, it's like, that point right here is like
[00:25:00.04] S3: It should be equal to 90, though, right?
[00:25:05.22] S4: It should be directed towards the center of the oval for it to move around the oval...
[00:25:08.24] Felix: I mentioned that same thing earlier. It's like, yeah, it's, at a curve, so would the circle be, whatever circle that curve, that arc made, so would it point to that center, or the center of the entire oval?
[00:25:24.03] S3: Let me look at the book
[00:25:26.22] S4: well based on the fact that 90 degrees is not actually towards the circle... it's just perpendicular to the tangent at a point
[00:25:36.05] Felix: the acceleration?
[00:25:36.19] S4: Yeah
[00:25:39.18] Felix: Oh, true, true
[00:25:37.19] S1: Yeah, I feel like no matter where you go on this entire oval, the acceleration vector and the other vector are always at 90 degrees
[00:25:49.12] S4: always equal to 90 degrees... because we already said the limit reaches 90 degrees
[00:25:56.21] S3: but this is different than the average acceleration vector which is the same direction as the change in velocity vector, right?
[00:26:01.18] S4: Yeah

Let's ask
[00:26:04.26] S4: Yeah, that's a good idea. But it says check with a...
[00:26:11.14] S1: Uh, what's our explanation?
[00:26:15.27] Felix: um, the velocity vector and the acceleration vector are always at 90 degrees
[00:26:19.29] S4: That's the limit, so that means...
[00:26:32.12] TA: Page 3 for you guys. Wow.
[00:26:34.25] S4: We have a question, though.
[00:26:36.12] TA: Yeah, what's up?
Oh wait
[00:26:38.15] S4: For,
[00:26:40.10] S1: (jokingly) check our reasoning. (TA laughs)
[00:26:41.00] S4: For, the one right before C - check your reasoning for part B with a Tutorial instructor - the last question we said always equal to 90 degrees?
[00:26:49.05] TA: (Looking at his answers) Yep,
[00:26:49.17] Felix: Oh, you have the answers?
[00:26:53.23] TA: Yeah, I have the answers. Well, I mean, I did it yesterday, basically. Um, yeah, OK. And what did you guys say for the limiting value on page 2?
[00:27:02.05] S4: The limiting value - 90 degrees.
[00:27:05.20] TA: OK. Cool. Yep, you guys are good.
[00:27:09.16] S3: So does that mean that the acceleration vector is always pointed towards the center of the circle?
[00:27:12.20] S4: Well, in an oval... in a circle, it's always pointing towards the center of the circle, but in an oval it's not.
[00:27:18.05] Felix: This would be, yeah, it would be, tangent, at a right angle to that vector of velocity // S4: At these points it's pointing towards the center
Yeah
[00:27:24.21] S4: But, I guess at this point, it's like there
[00:27:31.06] S4: So then when we travel in an elliptical plass... place, plass? Place around the sun,
[00:27:33.18] isn't the sun always at the same part and would always be pulling us towards where the sun is?
[00:27:40.27] TA: Yes
S4: So then what causes the elliptical... because when you're off that circular path then
[00:27:45.03] technically you're being pulled perpendicular to your path which would be a bit away from the sun
[00:27:49.20] Felix: Isn't the sun at one foci of the ellipse, and there's like, nothing in the other?
[00:27:53.16] TA: Yes. Yeah, the sun's at one foci, a random point is in the other. Umm,
[00:27:56.19] you would be right, if the earth moved at constant speed around the sun.
[00:27:59.28] S4: Yeah
TA: Remember equal areas, sweeps out equal areas in equal times // S4: Yeah, equal areas in equal times, so it's slower when it's closer and faster when it's... OK

[00:28:10.27] TA: I think that will come up later with Prof. U in lecture - you guys will look at the orbit of the earth
[00:28:17.14] S4: Kepler's Laws
[00:28:19.03] TA: Well, you'll touch on them briefly
[00:28:23.11] S4: So, so we have permission to move on to part C from the instructor
[00:28:25.13] TA: (Laughing) Yeah, you guys can go on - go for it
TRANSCRIPTION END

[00:28:32.24] S4 reads pg 3 C

[00:28:43.21] S4: The magnitude is greater at the new point than at point A
Felix: yeah
S4: magnitude is greater
Felix: because the change in velocity is greater

[00:29:06.21] S4: Change in velocity happens in less time

[00:29:42.26] S4: The change in direction of velocity happens in less time

[00:29:57.15] C2
S4: Point B, it's directed toward the center of the oval

[00:30:18.16] S3: Wait, so for part 2, is it directed to the center of the oval?
S4: At point B it is, because the tangent line...

[00:30:29.03] Felix: Right angle to the velocity

[00:30:59.06] D

[00:31:28.22] S4: Change in v remains the same, while change in t gets smaller, thus acceleration gets bigger

[00:32:42.06] E

[00:32:54.19] S3 and the others are still on D.
S3: If b is twice as long...
S1: does that make a twice as long as well?

[00:33:00.23] Felix: Yeah, because it's...

[00:33:17.12] Felix: Are you connecting them? Because one of them is speed and the other acceleration?

[00:33:55.06] S4: Acceleration has the same magnitude (for the top one)... acceleration has a different magnitude for the bottom one

[00:34:08.15] Senior TA: You guys are storming here. How's it going?

[00:36:43.10] S4: (Reading) Draw vectors to represent the velocity at two points on the track that are relatively close together. Label them points C and D.
[00:36:53.25] S4: (They discuss where to draw the diagrams)

***START
[00:38:00.12] S4: (Reading) "Label the angle between the head of vC and the tail of change in v. Is this angle"... it's greater
[00:38:09.01] than 90
S1: Yeah
[00:38:12.17] S4: So greater than 90 implies speeding up, less than 90 implies slowing down, and at 90 implies constant speed, (turns to look at TA) right?
[00:38:20.03] TA: (Laughs) Uh, in the limit... the absolute... yeah... or instantaneous
[00:38:29.28] S4: OK. Greater than 90 degrees.
[00:38:31.15] S4: OK. (Reading) "Determine the direction the average acceleration of the object between C and D" (Points to his drawing) There.
[00:38:43.24] S4: In a direction away from the center of the circle
[00:38:44.01] S1: yeah, what's up with that?
[00:38:46.28] S4: "Describe how you could use a limiting process to determine the direction of aC"
[00:38:55.13] S1: The instantaneous acceleration
[00:38:57.23] S4: The limit as D approaches C equals... as, final minus initial, vD - vC equals
[00:39:21.15] S3: Wait, for this real quick, this one here - is this the same direction as the change in velocity to determine the direction of the change in acceleration
[00:39:34.24] S3: You know what I'm saying?
S4: Hmm?
[00:39:35.20] S3: This here - is that the same direction as the change in velocity?
[00:39:43.13] S4: Yeah. Acceleration is change in velocity
S: Oh, yeah!
(S4 yawns)
[00:39:50.19] S3 (to Felix): Because my handwriting is better
[00:40:05.10] S3: So this is vC, like here, and then, vD, just the vector, it goes over here
[00:40:14.15] you know what I'm saying?
Felix: OK
[00:40:16.08] S3: Because if you were to add vC plus change in v, you'd get vD
[00:40:20.19] S4: And so the...
[00:40:22.20] Felix: So where'd it point?
[00:40:23.24] S3: It points just this way, like, this is this, this is this, this is the way it points here, so

[00:40:35.05] you know what I'm saying? It's greater than 90 degrees
[00:40:41.25] Felix: OK, so it doesn't point to the actual thing...
[00:40:43.18] S3: If they're the same, then it would be 90 degrees
S3: OK. Cool beans.
[00:40:54.20] Felix: It is pointing down?
S3: Yeah, but it says this one is longer because it's speeding up, so it's like pointing a little bit, like theta is greater than 90 degrees
[00:41:06.03] S1: Acceleration at C...
[00:41:14.10] S3: OK
S1: Um...
[00:41:24.01] S3: What's this limiting process thing in part C?
Felix: Yeah
S1: That one I don't really know
S1: Said the limit, as says D approaches C...
Felix: Yeah, that's what I put
[00:41:33.01] S4: (Reading) "Consider the acceleration at point C. Is the angle..." it's just, greater than
[00:41:37.15] wait, yeah, it's greater than 90.
[00:41:45.01] Because we say
S1: Is this at...
S4: that the acceleration is 180, the limit of it is 180
[00:41:51.14] If you approach it at ever increasing speed.
[00:41:57.02] S3: So this is what you said for here, and part B, and part C...
[00:41:59.03] S4: The limit of VD - vC as D goes to C is, I don't know if you can write that with correct notation, but angle between them approaches 180 degrees
[00:42:06.23] Felix: OK
[00:42:10.09] S4: Greater than 90 then
[00:42:13.26] S1: (Reading) Consider the acceleration...
[00:42:17.27] S1: So at this instantaneous point, the acceleration is not 90 degrees, it's greater than 90?
[00:42:24.06] S4: Yeah
[00:42:28.12] S3: So if we were to draw that here, right, so
[00:42:29.21] this point the acceleration is, like, just perpendicular to the velocity?
[00:42:34.09] S1: Well, he's saying it's greater, just a little bit. Yeah, like that.
[00:42:39.15] S3: Why is it greater? I don't really understand... I understand why the change would be greater, but I don't understand
[00:42:41.25] S4: Because the acceleration goes that way
S3: Yeah
[00:42:47.19] S4: OK, so then...
Felix: So
S4: So If you put them head to tail that's 180 degrees
[00:42:50.26] S3: Yeah. But what about the instantaneous acceleration here?
[00:42:51.27] S4: No, the instantaneous acceleration...
[00:42:54.12] Felix: But they're never at really the same spot...

[00:42:56.02] S4: that's what the limit is, our limit of calling it 180 degrees means that the instantaneous acceleration is 180 degrees, so therefore the acceleration is the same direction - 180 degrees. It's greater than 90.
[00:43:11.15] Felix: Wait... But, like, as it approaches it, wouldn't, this one's a little longer,
[00:43:15.20] S1: Draw the, at point C
[00:43:17.29] S4: Yeah, this one's a bit longer
[00:43:18.21] Felix: So it would be 90, it would approach 90
[00:43:20.20] S4: But you're making this way too... no, if it's longer,
S3: Yeah, if it's longer, it's going to be getting more and more obtuse
S4: 90 would be there. It's greater than 90.
[00:43:28.13] Look, see, look at this one (he draws). You choose points closer
[00:43:29.26] Felix: Yeah, but as it gets closer, the change in acceleration, so the change in the length of these gets smaller
[00:43:33.02] S4: Yes, but look at the drawing. Change in length does get smaller
[00:43:37.03] Felix: Yeah
S4: But it's approaching 180 degrees
[00:43:40.27] Felix: So wait, which angle are we talking about. Are we talking about the angle between these two things
[00:43:44.12] S4: No. First off, just what's the acceleration between these? Notice how
Felix: It's getting smaller
[00:43:47.15] S4: If you choose these two points. This is vD. These three are vD. The first one is that one. The second one is that one. The third one is that one. Change in v is acceleration. So as you can see, as this one is getting closer and closer, the acceleration approaches 180 degrees.
[00:44:05.10] Felix: Wait, I'm taking it as, we're finding this angle, is that wrong?
[00:44:10.04] S4: Yeah. We're not trying to find that angle.
Felix: OK, what are you finding?
[00:44:17.11] S4: If this is our first instant, and this is like an instant later (he draws)
[00:44:23.25] not an instant later, but, the acceleration is that way, right?
[00:44:28.06] and so if I ask you what is the angle between the acceleration and this, you'd say, what, like 150 degrees, something like that, right?
[00:44:35.23] Felix: OK, OK.
[00:44:36.22] S4: But at, it's, the limit is, the limit of this means 180 degrees. So, one increment of time later if that even exists, one increment of time, the smallest possible increment of time, the acceleration is 180 degrees. It's in the same direction as the movement.
[00:44:51.23] Felix: OK
[00:44:54.29] S4: The limit
[00:45:07.11] S3: That totally makes sense too, though, like
S1: Yeah, when you draw it out
[00:45:09.07] S3: Like if it was slowing down, then it would be negative acceleration, so it would be against the motion but if it stayed the same, it would be perpendicular because it would be zero

[00:45:18.12] S1: And if it was speeding up, it would go with the... gotcha
[00:45:20.17] S3: Yeah, since it's speeding up it acts in the same direction
S1: It just changes, so, you know, (inaudible) it's weird. It's strange.
***END

[00:45:42.25] S4: Well, student 3 is the most correct, because you approach 180 without ever actually reaching it

[00:46:21.16] S4: And student 3's reasoning is right, because if you did reach 180, it means you're not turning

[00:46:48.10] III.F
So it would be "less than"
S4: Yeah, it would approach 0 degrees

[00:47:43.25] S3: Wait, F is 90 degrees
S4: No, F is less than 90 degrees

[00:48:33.26] S3: You know, it's funny, it didn't make sense in the middle, but now it totally
Felix: Yeah, it was pretty rough in the middle

[00:50:54.08] TA: So what happens in C, I want to know what you think
S4: Well, the limit is 180 degrees, but it technically never reaches that, because that would that it's not turning

[00:51:20.22] TA: If vB is larger than vA, then delta v is never going to be 90
[00:51:30.27] So the acceleration vector has got to have two pieces to it - you're still moving on the curve, but you're moving up along the curve. So as a consequence you have to have one component that points inward and one component points along the curve, so you always have to have two pieces - one pushing you around the curve, and one speeding you up

[00:51:54.07] S3: If it were just speeding up, then it would just be one piece, right?

[00:52:27.26] S4: Much closer to 180
[00:52:27.26] TA: Right, and you can think about it in the following way - your change in velocity goes down if you're not speeding up very much, it gets closer to 90

[00:54:21.24] Felix: (Returns to do part C on the first page)

[00:55:45.14] S3: Is the average velocity just the change in velocity, or no?

[00:57:24.08] Felix: I think it does change, because it's going at less of a curve. It's not going through as great of an angle, so it would change

[00:58:03.02] Felix: The direction gets closer to the direction of A as these two get smaller

[00:58:21.03] S3: It's not changing speed, it's just direction. So you're thinking it gets smaller?

[01:01:09.29] They have left - I zoom in on Mel's group