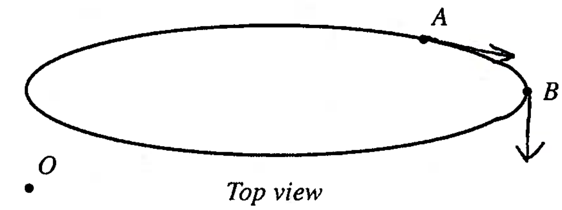
MOTION IN TWO DIMENSIONS

I. Velocity



An object is moving clockwise around an oval track as shown at right. Sketch the trajectory on a large sheet of paper. (Make your diagram *large*.) Point *O* is the origin of the coordinate system.

A. Draw $\vec{r}_A$ and $\vec{r}_B$, the position vectors for the object when it is at points *A* and *B*.

B. Draw the vector that represents the displacement of the object (*i.e.*, the change in position) from *A* to *B*.

Describe how to use the displacement vector to determine the direction of the average velocity of the object between *A* and *B*. Draw a vector to represent the average velocity.

C. Choose a point on the oval between points *A* and *B*, and label that point *B′*.

As point *B′* is chosen to lie closer and closer to point *A*, does the direction of the average velocity over the interval *AB′* change? If so, how?

The average velocity becomes closer to the tangent at point A

What happens to the magnitude of the displacement as point *B′* is chosen to lie closer and closer to point *A?* Does this magnitude approach a limiting value? If so, what is that value?

It gets smaller. Zero

Must the magnitude of the average velocity change in the same way? Explain.

Drives at the same speed throughout

D. Describe the direction of the (instantaneous) velocity of the object at point *A*.

Tangential

How would you characterize the direction of the (instantaneous) velocity at *any* point on the trajectory?

Tangent to that point on the oval

II. Acceleration for motion with constant speed

Suppose that the object in section I is moving around the track at *constant speed*.

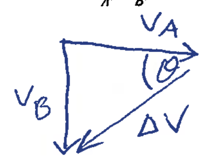
A. On your large diagram, draw vectors to represent the velocities at points *A* and *B*.

1. Did the *speed* of the object change? Explain how you can tell from the vectors that you have drawn. Magnitude of the velocity is constant

2. Did the *velocity* of the object change? Explain how you can tell from the vectors that you have drawn. Yes - change direction in velocity did change

B. On a *separate* part of your paper, copy the velocity vectors $\vec{v}_A$ and $\vec{v}_B$ so that their "tails" are at the same point.

$\vec{v}_A$, $\vec{v}_B$, and $\Delta\vec{v}$



1. From your diagram, determine the *change-in-velocity vector*, $\Delta\vec{v}$.

2. Describe how to use the change-in-velocity vector to determine the direction of the average acceleration of the object between *A* and *B*. Draw a vector to represent the average acceleration between points *A* and *B*.
Change in velocity vector is in the same direction as acceleration

3. On your diagram, label the angle θ between the "head" of $\vec{v}_A$ and the "tail" of $\Delta\vec{v}$. Is this angle *greater than, less than*, or *equal to* 90°?

As point *B* is chosen to lie closer and closer to point *A*, does the angle θ *increase*, *decrease*, or *remain the same?* Explain how you can tell.
Increases

Does the angle θ approach a *limiting* value? If so, what is that value?
~~180°~~ 90°

4. As point *B* is chosen to lie closer and closer to point *A*, the magnitude of $\Delta\vec{v}$ approaches zero. Does the magnitude of the average acceleration also approach zero? Explain.
~~No - there is always acceleration~~
Yes b/c $\Delta V \to 0$

5. Determine the direction of the (instantaneous) acceleration ~~at~~ point *A*.
~~Towards the center of the oval~~
Normal to the velocity

Draw a diagram that shows the velocity and acceleration at point *A*. Place both vectors with their tails at point *A*. Is the angle between the acceleration and the velocity *greater than*, *less than*, or *equal to* 90°?

Equal to 90°

➪ Check your reasoning for part B with a tutorial instructor.

C. Suppose you were to choose a new point on the trajectory where the curve is tighter than at point *A* (*e.g.*, point *B*).

1. Is the magnitude of the acceleration at the new point *greater than*, *less than*, or *equal to* the magnitude of the acceleration at point *A?* Explain your reasoning.

   Greater - change in velocity is greater

   Change in direction of velocity happens in less time

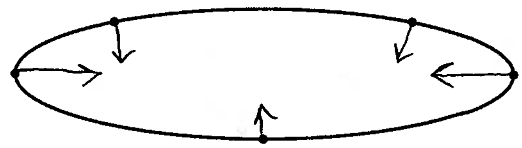
2. Describe the direction of the acceleration at the new point.

   At a right angle to the velocity

3. At each of the indicated points on the diagram at right, sketch a vector that represents the acceleration of the object.

   Is the acceleration directed toward the "center" of the oval at every point on the trajectory?

Top view



Acceleration vectors for constant speed

D. Suppose that the object had been moving faster (*e.g.*, twice as fast), but still uniformly.

Would the magnitude of the acceleration at a particular point (*e.g.*, point *A*) be *greater than*, *less than*, or *equal to* that when the object was moving more slowly? Draw a diagram to support your answer. Greater than, b/c change in v is same while change in t gets smaller

E. Summarize your results from parts C and D above in a rule for comparing the magnitudes of the accelerations of two objects when:

- the speeds of the two objects are the same but the curvatures of the paths are different

  Magnitude of acceleration is the same

- the curvatures of the paths are the same but the speeds of the two objects are different

  Magnitude of acceleration is different

➪ Discuss parts C–E with a tutorial instructor.

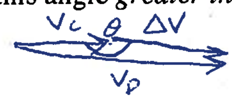
III. Acceleration for motion with increasing speed

Suppose that the object in section II is *speeding up* as it moves around the oval track.

A. Draw vectors to represent the velocity at two points on the track that are relatively close together. Label the two points *C* and *D*.

B. On a *separate* part of your paper, copy the velocity vectors $\vec{v}_C$ and $\vec{v}_D$ so that they are tail-to-tail. Use the same procedure as in previous sections to determine the change in velocity, $\Delta\vec{v}$.

Label the angle θ between the head of $\vec{v}_C$ and the tail of $\Delta\vec{v}$. Is this angle *greater than, less than*, or *equal to* 90°?



Greater than 90°

Determine the direction of the average acceleration of the object between *C* and *D*.

Same as ΔV - away from center of circle

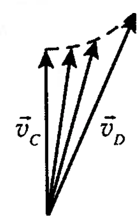
C. Describe how you could use a limiting process to determine the direction of $\vec{a}_C$, the (instantaneous) acceleration of the object at point *C*.

$V_D - V_C$ = angle approaches 180°

Consider the acceleration at point *C*. Is the angle θ between the head of $\vec{v}_C$ and the tail of $\vec{a}_C$ *greater than, less than*, or *equal to* 90°?

Greater than 90

D. Three students consider how θ changes as point *D* is chosen to lie closer and closer to point *C*.



Student 1's velocity vectors

Student 1: "The object is speeding up and moving on a curve, so $\vec{v}_C$ and $\vec{v}_D$ differ in both magnitude and direction. But, as point D gets close to C, they eventually have the same magnitude like I've drawn. So, in the limit, the angle θ between the head of $\vec{v}_C$ and the tail of $\Delta\vec{v}$ is 90°."

Student 2: "But that's the angle that we got when the speed was constant. Since the object is speeding up, I think it should be 180°"

Student 3: "Well the angle can't be 180° because then the object would just be speeding up and not turning. Maybe the angle is something between 90° and 180°."

With which student, if any, do you agree? Explain your reasoning.

S3 is closest to being correct - it approaches 180° but never actually gets there

E. Are your answers to parts C and D consistent? If not, resolve the inconsistency.

F. When an object is moving on a curve and speeding up, is the angle between $\vec{v}$ and $\vec{a}$ placed *tail-to-tail* (not head-to-tail as above) *greater than, less than*, or *equal to* 90°?

Less than; 0° is limit, but it never reaches it

➪ Discuss your results from section III with a tutorial instructor.